

# Supersymmetry Breaking Scalar Masses and Trilinear Soft Terms From High-Dimensional Operators in $E_6$ SUSY GUT

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## Abstract

In the GmSUGRA scenario with the higher-dimensional operators containing the GUT Higgs fields, we systematically studied the supersymmetry breaking scalar masses, SM fermion Yukawa coupling terms, and trilinear soft terms in the  $E_6$  SUSY GUT model where the gauge symmetry is broken down to the  $SO(10) \times U(1)$  gauge symmetry,  $SU(3)_C \times SU(3)_L \times SU(3)_R$  gauge symmetry,  $SU(6) \times SU(2)_a$  ( $a = L, R, X$ ) gauge symmetry, flipped  $SU(5)$  gauge symmetry etc. In addition, we considered the scalar and gaugino mass relations, which can be preserved from the GUT scale to the electroweak scale under one-loop RGE running, in the  $SU(3)_C \times SU(3)_L \times SU(3)_R$  model arising from the  $E_6$  model. With such relations, we may distinguish the mSUGRA and GmSUGRA scenarios if we can measure the supersymmetric particle spectrum at the LHC and ILC.

*Keywords:* Higher dimensional operator;  $E_6$  SUSY GUT; supersymmetry; mass relations.

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## 1. Introduction

Supersymmetry naturally solves the gauge hierarchy problem of the Standard Model (SM). The unification of the three gauge couplings  $SU(3)_C$ ,  $SU(2)_L$  and  $U(1)_Y$  in the supersymmetric Standard Model at about  $2 \times 10^{16}$  GeV [1] strongly suggests the existence of Grand Unified Theories (GUTs). In addition, GUT models such as  $SU(5)$  [2],  $SO(10)$  [3], and superstring-inspired  $E_6$  [4, 5] models etc [6] give us deep insights into the other SM problems such as the emergence of the fundamental forces, the assignments and quantization

of their charges, the fermion masses and mixings, and beyond. Although supersymmetric GUTs are attractive it is challenging to test them at the Large Hadron Collider (LHC), the future International Linear Collider (ILC), and other experiments.

In traditional supersymmetric SMs, supersymmetry is broken in the hidden sector and the supersymmetry breaking effects can be mediated to the observable sector via gravity [7], gauge interactions [8, 9], or super-Weyl anomaly [10, 11, 12], or other mechanisms. Recently, considering GUTs with higher-dimensional operators [8, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24] and F-theory GUTs with  $U(1)$  fluxes [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35], generalized mSUGRA (GmSUGRA) scenario is proposed [36] in which the gaugino mass relations are studied and their indices are defined. In our previous works [37, 38], we discuss (in the context of GmSUGRA) the supersymmetry breaking scalar masses and trilinear soft terms in  $SU(5)$  and  $SO(10)$  GUT models with various higher dimensional Higgs fields. It is also interesting to discuss in  $E_6$  SUSY GUT model the supersymmetry breaking scalar masses and trilinear soft terms from non-renormalizable Kahler potential and non-renormalizable superpotential.

The exceptional group  $E_6$  has been proposed as an attractive unification group with several desirable features: 1)  $E_6$  was the next natural anomaly-free choice for a GUT group after  $SO(10)$ ; 2) all the basic fermions of one generation belong to a single irreducible representation **27**. We know that within the context of heterotic superstring theory in ten dimensions, gauge and gravitational anomaly cancelation was found to occur only for the gauge groups  $SO(32)$  or  $E_8 \times E_8$  [39]. Compactification on a Calabi-Yau manifold with an  $SU(3)$  holonomy results in the breaking  $E_8 \rightarrow SU(3) \times E_6$  with the  $SU(3)$  gauge field becoming the spin connection on the compactified space. This result inspired the current interests in  $E_6$  GUT [40, 41, 42, 43, 44, 45, 46].

In this paper, we consider the supersymmetry breaking scalar masses and trilinear soft terms from non-renormalizable Kahler potential and non-renormalizable superpotential in  $E_6$  SUSY GUT. We systematically calculate the supersymmetry breaking scalar masses, SM fermion Yukawa coupling terms, and trilinear soft terms in  $E_6$  models where the gauge symmetry is broken down to the  $SO(10) \times U(1)$  gauge symmetry, flipped  $SO(10)$  [50, 51, 52, 53] gauge symmetry,  $SU(3)_C \times SU(3)_L \times SU(3)_R$  gauge symmetry,  $SU(6) \times SU(2)_X$  [54, 55, 56] gauge symmetry, flipped  $SU(5) \times U(1)_X$  gauge symmetry [47, 48, 49]. We should note that we investigate in this work only the group-theoretical necessities for such a breaking, but no dynamical model is

constructed to give the symmetry-breaking vacuum expectation value (VEV). Besides, in our work we consider basically one single spontaneously symmetry breaking step for  $E_6$ . As a result, no investigations about the running of the coupling in general and possible constraints from perturbativity have been made in this work. We examine the scalar and gaugino mass relations, which are valid from the GUT scale to the electroweak scale under one-loop renormalization group running in the  $SU(3)_C \times SU(3)_L \times SU(3)_R$  models arising from the  $E_6$  GUT model. With these relations, we may distinguish the mSUGRA and GmSUGRA scenarios if the supersymmetric particle spectrum can be measured at the LHC and ILC.

This paper is organized as follows. In Section 2, we briefly review four-dimensional  $E_6$  GUTs and its symmetry breaking chains. In Section 3, we explain the general gravity mediated supersymmetry breaking. We derive the scalar masses in Section 4, and the SM fermion Yukawa coupling terms and trilinear soft terms in Section 5. In Section 6 we consider the scalar and gaugino mass relations. Section 7 contains our conclusions.

## 2. Brief Review of Grand Unified Theories

In this Section we explain our conventions. In supersymmetric SMs, we denote the left-handed quark doublets, right-handed up-type quarks, right-handed down-type quarks, left-handed lepton doublets, right-handed neutrinos and right-handed charged leptons as  $Q_L^i$ ,  $(U_L^c)^i$ ,  $(D_L^c)^i$ ,  $L_L^i$ ,  $(N_L^c)^i$ , and  $(E_L^c)^i$ , respectively. Also, we denote one pair of Higgs doublets as  $h_u$  and  $h_d$ , which give masses to the up-type quarks/neutrinos and the down-type quarks/charged leptons, respectively.

First, we briefly review the  $E_6$  GUT model.  $E_6$  can break into gauge group  $SU(3)_C \times SU(3)_L \times SU(3)_R$ ,  $SU(6) \times SU(2)_a$  (a=L,R,X) gauge symmetry,  $SO(10)$  gauge symmetry, flipped  $SO(10)$  gauge symmetry, flipped  $SU(5)$  gauge symmetry, Pati-Salam  $SU(4)_c \times SU(2)_L \times SU(2)_R$  gauge symmetry,  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_1 \times U(1)_2$  gauge symmetry. Each generation of standard model matter contents are filled into **27** dimensional representations of  $E_6$  GUT group. Depending on different gauge symmetry breaking chains, the standard model matter contents are filled differently.

- $SU(3)_C \times SU(3)_L \times SU(3)_R$

Under  $SU(3)_C \times SU(3)_L \times SU(3)_R$  gauge symmetry, the **27** and **78**

dimensional representation of  $E_6$  are decomposed [65]

$$\mathbf{27} = (\mathbf{3}, \mathbf{3}, \mathbf{1}) \oplus (\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}) \oplus (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}), \quad (1)$$

$$\begin{aligned} \mathbf{78} = & (\mathbf{8}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{8}) \\ & \oplus (\mathbf{3}, \bar{\mathbf{3}}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3}, \mathbf{3}). \end{aligned} \quad (2)$$

The filling of the standard model matter contents in terms of gauge group  $SU(3)_C \times SU(3)_L \times SU(3)_R$

$$\begin{aligned} X_L^a(\mathbf{3}, \mathbf{3}, \mathbf{1}) & \sim \begin{pmatrix} u_L \\ d_L \\ D_L \end{pmatrix}, \quad (X_L^c)^a(\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}) \sim \begin{pmatrix} u_L^c \\ d_L^c \\ D_L^c \end{pmatrix}, \\ N^a(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) & \sim \begin{pmatrix} H_1^0 & H_2^+ & e_L^c \\ H_1^- & H_2^0 & -\nu_L^c \\ e_L & -\nu_L & n_0 \end{pmatrix}, \end{aligned} \quad (3)$$

where  $(a = 1, 2, 3)$  for three families. The breaking of gauge group  $E_6$  into  $SU(3)_C \times SU(3)_L \times SU(3)_R$  is achieved by **650** dimensional Higgs field.<sup>1</sup> To break gauge symmetry  $SU(3)_C \times SU(3)_L \times SU(3)_R$  into its subgroup  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_1 \times U(1)_2$ , we can use **650** dimensional Higgs fields to acquire  $(\mathbf{1}, \mathbf{8}, \mathbf{8})$  term VEVs. It is also possible to use **27**,  **$\bar{27}$**  dimensional representation Higgs to achieve the second stage symmetry breaking into the left-right gauge group  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ .

- $SO(10) \times U(1)$

The fundamental representation **27** and adjoint representation **78** of  $E_6$  can be decomposed in term of  $SO(10) \times U(1)$

$$\mathbf{27} = \mathbf{16}_1 \oplus \mathbf{10}_{-2} \oplus \mathbf{1}_4, \quad (5)$$

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<sup>1</sup>The decomposition of **650** dimensional Higgs into  $SU(3)_C \times SU(3)_L \times SU(3)_R$  quantum numbers is

$$\begin{aligned} \mathbf{650} = & (\mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{8}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{8}) \oplus (\bar{\mathbf{3}}, \mathbf{3}, \mathbf{3}) \\ & \oplus (\bar{\mathbf{3}}, \mathbf{3}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}}, \bar{\mathbf{3}}) \oplus (\mathbf{3}, \bar{\mathbf{3}}, \bar{\mathbf{3}}) \oplus (\mathbf{3}, \mathbf{6}, \bar{\mathbf{3}}) \oplus (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{6}) \oplus (\bar{\mathbf{3}}, \bar{\mathbf{6}}, \mathbf{3}) \\ & \oplus (\bar{\mathbf{3}}, \mathbf{3}, \bar{\mathbf{6}}) \oplus (\bar{\mathbf{6}}, \bar{\mathbf{3}}, \bar{\mathbf{3}}) \oplus (\mathbf{6}, \mathbf{3}, \mathbf{3}) \oplus (\mathbf{8}, \mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8}, \mathbf{8}) \\ & \oplus (\mathbf{8}, \mathbf{1}, \mathbf{8}). \end{aligned} \quad (4)$$

$$\mathbf{78} = \mathbf{45}_0 \oplus \mathbf{16}_{-3} \oplus \overline{\mathbf{16}}_3 \oplus \mathbf{1}_0 . \quad (6)$$

The filling of standard model contents is different between the flipped  $SO(10)$  scenario and the  $U(1)$  extension of ordinary  $SO(10)$  scenario.

–  $U(1)$  extension of ordinary  $SO(10)$

In this scenario, the standard model matter contents are filled in  $\mathbf{16}$  dimensional spinor representation (decomposed in Georgi-Glashow  $SU(5) \times U(1)$ )

$$\begin{aligned} \mathbf{16}_1 &= (\mathbf{10}_{Q_L, U_L^c, E_L^c}, \bar{\mathbf{5}}_{D_L^c, L_L}, \mathbf{1}_{N_L^c}) , \\ \mathbf{10}_{-2} &= (\mathbf{5}_H, \bar{\mathbf{5}}_H) , \\ \mathbf{1}_{-4} &= \mathbf{1}_S . \end{aligned} \quad (7)$$

Flipped  $SU(5) \times U(1)_X$  model [47, 48, 49] can also be embedded into  $SO(10)$ .

In this scenario, the standard model matter contents are filled as

$$\begin{aligned} \mathbf{16}_1 &= (\mathbf{10}_{Q_L, D_L^c, N_L^c}, \bar{\mathbf{5}}_{U_L^c, L_L}, \mathbf{1}_{E_L^c}) \\ \mathbf{10}_{-2} &= (\mathbf{5}_V, \bar{\mathbf{5}}_V) , \\ \mathbf{1}_{-4} &= \mathbf{1}_S . \end{aligned} \quad (8)$$

To break the flipped  $SU(5)$  GUT and electroweak gauge symmetries, we introduce two pairs of Higgs fields whose quantum numbers under  $SU(5) \times U(1)_X$  are

$$H = (\mathbf{10}, \mathbf{1}) , \quad \bar{H} = (\overline{\mathbf{10}}, -\mathbf{1}) , \quad h = (\mathbf{5}, -\mathbf{2}) , \quad \bar{h} = (\bar{\mathbf{5}}, \mathbf{2}) , \quad (9)$$

where  $h$  and  $\bar{h}$  contain the Higgs doublets  $h_d$  and  $h_u$ , respectively. In flipped  $SU(5)$ , Doublet-Triplet splitting problems can be solved via the elegant missing partner mechanism. This mechanism is however spoiled if we embed flipped  $SU(5)$  into  $SO(10)$  [57, 58, 59].

– Flipped  $SO(10)$

Flipped  $SO(10)$  is introduced in [50, 51, 52, 53] to keep the elegant missing partner mechanism when embedding flipped  $SU(5)$  into  $SO(10) \times U(1)$ . In this scenario, the standard model matter

contents (with extra exotic particles) are filled as

$$\mathbf{16_1} = (\mathbf{10}_{Q_L, D_L^c, N_L^c}, \bar{\mathbf{5}}_{\mathbf{V}}, \mathbf{1}_{\mathbf{S}}), \quad (10)$$

$$\mathbf{10_{-2}} = (\mathbf{5}_{\mathbf{V}}, \bar{\mathbf{5}}_{U_L^c, L_L}), \quad (11)$$

$$\mathbf{1_{-4}} = \mathbf{1}_{E_L^c}, \quad (12)$$

in which we flip  $\bar{\mathbf{5}}_{U_L^c, L_L}$  with  $\bar{\mathbf{5}}_{\mathbf{V}}$ ,  $\mathbf{1}_{E_L^c}$  with  $\mathbf{1}_{\mathbf{S}}$  with respect to ordinary embedding of flipped  $SU(5)$  into  $SO(10)$ .

- $SU(6) \times SU(2)_a (a=L, R, X)$

In the simplest grand unifying group  $SU(5)$ , natural implementation of doublet triplet splitting seems to require the use of the relatively large representations  $\mathbf{50}$ ,  $\bar{\mathbf{50}}$  and  $\mathbf{75}$ . The search for a simpler solution has lead various authors to consider the extension of the gauge symmetry to  $SU(6)$  [54, 55, 56] which allows for more possibilities: (i) The light Higgs doublets emerge as the pseudo-Goldstone bosons of a broken global symmetry of the superpotential [60, 61]. (ii) the sliding singlet mechanism where the desired VEV pattern follows automatically from the conditions of the supersymmetric minima condition. In the context of  $SU(5)$  model, there are severe difficulties due to radiative corrections which actually lift the MSSM doublet masses to an intermediate scale. If instead one considers an embedding of the  $SU(5)$  model into the  $SU(6)$  group, then the problems associated with radiative instability of doublet-triplet splitting can be cured [62, 63].

The fundamental representation  $\mathbf{27}$  and adjoint representation  $\mathbf{78}$  of  $E_6$  can be decomposed in term of  $SU(6) \times SU(2)$

$$\mathbf{27} = (\bar{\mathbf{6}}, \mathbf{2}) \oplus (\mathbf{15}, \mathbf{1}), \quad (13)$$

$$\mathbf{78} = (\mathbf{35}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}) \oplus (\mathbf{20}, \mathbf{2}). \quad (14)$$

The breaking of  $E_6$  into  $SU(6) \times SU(2)_a (a=L, R, X)$  can be achieved by  $\mathbf{650}$  dimensional Higgs field whose decomposition reads

$$\begin{aligned} \mathbf{650} = & (\mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{35}) \oplus (\mathbf{2}, \mathbf{20}) \oplus (\mathbf{3}, \mathbf{35}) \oplus (\mathbf{2}, \mathbf{70}) \\ & \oplus (\mathbf{2}, \bar{\mathbf{70}}) \oplus (\mathbf{1}, \mathbf{189}). \end{aligned} \quad (15)$$

Different choice of the  $SU(2)_a$  leads to different filling of the standard model matter contents

- $E_6 \rightarrow SU(6) \times SU(2)_X \rightarrow SU(5) \times U(1) \times SU(2)_X$ :

The decomposition of **27** representation in terms of  $SU(5) \times SU(2)_X$

$$(\mathbf{15}, \mathbf{1}) = (\mathbf{10}, \mathbf{1}) \oplus (\mathbf{5}, \mathbf{1}), \quad (16)$$

$$(\bar{\mathbf{6}}, \mathbf{2}) = (\bar{\mathbf{5}}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{2}). \quad (17)$$

We identify the matter contents

$$(\mathbf{10}, \mathbf{1}) \supset (U_L, U_L^c, D_L, E_L^c), \quad (18)$$

$$(\bar{\mathbf{5}}, \mathbf{2}) \supset (D_L^c, E_L, N_L), \quad (19)$$

$$(\mathbf{1}, \mathbf{2}) \supset N_L^c. \quad (20)$$

- $E_6 \rightarrow SU(6) \times SU(2)_L \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1)_2$ :

This symmetry breaking chain was proposed in [64]. The decomposition of **27** representation of  $E_6$  in terms of gauge group  $SU(4)_c \times SU(2)_R \times SU(2)_L$  reads

$$(\mathbf{15}, \mathbf{1}) = (\mathbf{6}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{2}, \mathbf{1}), \quad (21)$$

$$(\bar{\mathbf{6}}, \mathbf{2}) = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \oplus (\mathbf{1}, \bar{\mathbf{2}}, \mathbf{2}). \quad (22)$$

We identify the matter contents

$$(\mathbf{4}, \mathbf{2}, \mathbf{1}) \supset (U_R, D_R, E_R, N_R), \quad (23)$$

$$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \supset (U_R^c, D_R^c, E_R^c, N_R^c). \quad (24)$$

- $E_6 \rightarrow SU(6) \times SU(2)_R \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1)_2$ :

The decomposition of **27** representation in terms of gauge group  $SU(4)_c \times SU(2)_L \times SU(2)_R$  reads

$$(\mathbf{15}, \mathbf{1}) = (\mathbf{6}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{2}, \mathbf{1}), \quad (25)$$

$$(\bar{\mathbf{6}}, \mathbf{2}) = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \oplus (\mathbf{1}, \bar{\mathbf{2}}, \mathbf{2}). \quad (26)$$

We identify the matter contents

$$(\mathbf{4}, \mathbf{2}, \mathbf{1}) \supset (U_L, D_L, e_L, \nu_L), \quad (27)$$

$$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \supset (U_L^c, D_L^c, e_L^c, \nu_L^c). \quad (28)$$

### 3. General Gravity Mediated Supersymmetry Breaking

The supegravity scalar potential can be written as [7]

$$V = M_*^4 e^G [G^i (G^{-1})_i^j G_j - 3] + \frac{1}{2} \text{Re} [(f^{-1})_{ab} \hat{D}^a \hat{D}^b] , \quad (29)$$

where  $M_*$  is the fundamental scale, D-terms are

$$\hat{D}^a \equiv -G^i (T^a)_i^j \phi_j = -\phi^{j*} (T^a)_j^i G_i , \quad (30)$$

and the Kähler function  $G$  as well as its derivatives and metric  $G_i^j$  are

$$G \equiv \frac{K}{M_*^2} + \ln \left( \frac{W}{M_*^3} \right) + \ln \left( \frac{W^*}{M_*^3} \right) , \quad (31)$$

$$G^i = \frac{\delta G}{\delta \phi_i} , \quad G_i = \frac{\delta G}{\delta \phi_i^*} , \quad G_i^j = \frac{\delta^2 G}{\delta \phi_i^* \delta \phi_j} , \quad (32)$$

where  $K$  is Kähler potential and  $W$  is superpotential.

Because the gaugino masses have been studied previously [36], we only consider the supersymmetry breaking scalar masses and trilinear soft terms in this paper. To break supersymmetry, we introduce a chiral superfield  $S$  in the hidden sector whose  $F$  term acquires a vacuum expectation value (VEV), *i.e.*,  $\langle S \rangle = \theta^2 F_S$ . To calculate the scalar masses and trilinear soft terms, we consider the following superpotential and Kähler potential

$$W = \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k + \alpha \frac{S}{M_*} \left( \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k \right) , \quad (33)$$

$$K = \phi_i^\dagger \phi_i + \beta \frac{S^\dagger S}{M_*^2} \phi_i^\dagger \phi_i , \quad (34)$$

where  $y^{ijk}$ ,  $\alpha$ , and  $\beta$  are Yukawa couplings. Thus, we obtain the universal supersymmetry breaking scalar mass  $m_0$  and trilinear soft term  $A$  of mSUGRA

$$m_0^2 = \beta \frac{|F_S|^2}{M_*^2} , \quad A_{ijk} = A_0 y_{ijk} = \left( \alpha \frac{F_S}{M_*} \right) y_{ijk} . \quad (35)$$

When we break the GUT gauge symmetry by giving VEV to the Higgs field  $\Phi$ , we can have the general superpotential and Kähler potential

$$\begin{aligned} W = & \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k + \frac{1}{6} \left( h^{ijk} \frac{\Phi}{M_*} \phi_i \phi_j \phi_k \right) + \alpha \frac{S}{M_*} \left( \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k \right) \\ & + \alpha' \frac{T}{M_*} \left( \frac{1}{6} y^{ijk} \frac{\Phi}{M_*} \phi_i \phi_j \phi_k \right) , \end{aligned} \quad (36)$$



$$\begin{aligned}
K = & \phi_i^\dagger \phi_i + \frac{1}{2} h' \phi_i^\dagger \left( \frac{\Phi}{M_*} + \frac{\Phi^\dagger}{M_*} \right) \phi_i + \beta \frac{S^\dagger S}{M_*^2} \phi_i^\dagger \phi_i \\
& + \frac{1}{2} \beta' \Phi \frac{T^\dagger T}{M_*^2} \phi_i^\dagger \left( \frac{\Phi}{M_*} + \frac{\Phi^\dagger}{M_*} \right) \phi_i ,
\end{aligned} \tag{37}$$

where  $h^{ijk}$ ,  $\alpha'$ ,  $\beta'^\Phi$  and  $h'$  are Yukawa couplings, and  $T$  can be  $S$  or another chiral superfield with non-zero  $F$  term, *i.e.*,  $\langle T \rangle = \theta^2 F_T$ . Therefore, after the GUT gauge symmetry is broken by the VEV of  $\Phi$ , we obtain the non-universal supersymmetry breaking scalar masses and trilinear soft terms, which will be studied in the following. For simplicity, we assume  $h' = 0$  in the following discussions since we can redefine the fields and the SM fermion Yukawa couplings.

#### 4. Non-Universal Soft masses for sfermions in $E_6$ SUSY GUT

We know that the matter contents in  $E_6$  GUT are fitted into **27** dimensional representations. Thus the non-minimal kinetic terms for matter contents in  $E_6$  GUT requires the group tensor production decomposition [65]

$$\overline{\mathbf{27}} \otimes \mathbf{27} = \mathbf{1} \oplus \mathbf{78} \oplus \mathbf{650} . \tag{38}$$

So in order to construct general gauge invariant non-renormalizable Kahler potential terms, we need to consider Higgs in **78** and **650** dimensional representations.

##### 4.1. $E_6$ To $SO(10) \times U(1)_1$ Model

The fundamental representation **27** and adjoint representation **78** of  $E_6$  can be decomposed in term of  $SO(10) \times U(1)$

$$\mathbf{27} = \mathbf{16}_1 \oplus \mathbf{10}_{-2} \oplus \mathbf{1}_4 , \tag{39}$$

$$\mathbf{78} = \mathbf{45}_0 \oplus \mathbf{16}_{-3} \oplus \overline{\mathbf{16}}_3 \oplus \mathbf{1}_0 . \tag{40}$$

The **78** dimensional representation Higgs can acquire Vacuum Expectation Values (VEVs) which break  $E_6$  into  $SO(10) \times U(1)$ . Such VEVs can be written as  $27 \times 27$  matrix as follows

$$\langle \Phi \rangle^{\mathbf{78}} = \frac{\hat{v}_{\mathbf{78}}}{2\sqrt{6}} \text{diag}(\underbrace{1, \dots, 1}_{16}, \underbrace{-2, \dots, -2}_{10}, 4) . \tag{41}$$

with normalization factor

$$c = Tr(< \Phi >^2) = T(\mathbf{27}) = 3 .$$

We normalize the VEVs with  $Tr(T^a T^b) = T(r)\delta^{ab}$ , so that same results will be obtained when the same VEVs are written as different  $n \times n$  matrix forms.

The **650** dimensional Higgs can also acquire Vacuum Expectation Values (VEVs) which break  $E_6$  into  $SO(10) \times U(1)$ . Such VEVs can be written as  $27 \times 27$  matrix as follows

$$< \Phi >^{\mathbf{650}} = \frac{\hat{v}_{\mathbf{650}}}{12\sqrt{5}} \text{diag}(\underbrace{-5, \dots, -5}_{16}, \underbrace{4, \dots, 4}_{10}, 40) . \quad (42)$$

with normalization factor  $c = 3$ .

There are two possible ways to fill the matter contents into  $SO(10) \times U(1)$ .

- $U(1)$  Extension of Ordinary  $SO(10)$ :

In this scenario, the Standard Model matter contents can be filled into  $\mathbf{16}_1$  representation within  $\mathbf{27}$ . After **78** dimensional Higgs acquire VEVs, all the sfermions acquire masses

$$m_{\tilde{f}}^2 = (m_0^U)^2 + \frac{\hat{v}_{\mathbf{78}}}{2\sqrt{6}M_*} \beta'^{\mathbf{78}} (m_0^N)^2. \quad (43)$$

Here and in the following sections, we define the universal part for soft sfermion masses

$$(m_0^U)^2 = \frac{\beta}{M_*^2} F_S^* F_S , \quad (44)$$

and mass parameters within non-universal part for soft sfermion masses

$$(m_0^N)^2 = \frac{1}{2M_*^2} F_T^* F_T . \quad (45)$$

After **650** dimensional Higgs acquire VEVs, all the sfermions acquire masses

$$m_{\tilde{f}}^2 = (m_0^U)^2 - \frac{5\hat{v}_{\mathbf{650}}}{12\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2. \quad (46)$$

- Flipped  $SO(10)$ :

In this scenario, the matter contents are filled as (in notation of  $SU(5)$ )

$$\mathbf{16_1} = (\mathbf{10}_{Q,D_L^c,N_L^c}, \bar{\mathbf{5}}_{\mathbf{V}}, \mathbf{1}_{\mathbf{V}}) , \quad (47)$$

$$\mathbf{10_{-2}} = (\mathbf{5}_{\mathbf{V}}, \bar{\mathbf{5}}_{U_L^c,L_L}) , \quad (48)$$

$$\mathbf{1_{-4}} = \mathbf{1}_{E_L^c} . \quad (49)$$

After **78** dimensional Higgs acquire VEVs, the sfermions acquire masses

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2 + \frac{\hat{v}_{78}}{2\sqrt{6}M_*} \beta'^{78} (m_0^N)^2 , \quad (50)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2 - \frac{\hat{v}_{78}}{\sqrt{6}M_*} \beta'^{78} (m_0^N)^2 , \quad (51)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2 + \frac{\hat{v}_{78}}{2\sqrt{6}M_*} \beta'^{78} (m_0^N)^2 , \quad (52)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2 - \frac{\hat{v}_{78}}{\sqrt{6}M_*} \beta'^{78} (m_0^N)^2 , \quad (53)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2 + 2 \frac{\hat{v}_{78}}{\sqrt{6}M_*} \beta'^{78} (m_0^N)^2 . \quad (54)$$

After **650** dimensional Higgs acquire VEVs, the sfermions acquire masses

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2 - 5 \frac{\hat{v}_{650}}{12\sqrt{5}M_*} \beta'^{650} (m_0^N)^2 , \quad (55)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2 + \frac{\hat{v}_{650}}{3\sqrt{5}M_*} \beta'^{650} (m_0^N)^2 , \quad (56)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2 - 5 \frac{\hat{v}_{650}}{12\sqrt{5}M_*} \beta'^{650} (m_0^N)^2 , \quad (57)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2 + \frac{\hat{v}_{650}}{3\sqrt{5}M_*} \beta'^{650} (m_0^N)^2 , \quad (58)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2 + 10 \frac{\hat{v}_{650}}{3\sqrt{5}M_*} \beta'^{650} (m_0^N)^2 . \quad (59)$$

#### 4.2. $E_6$ To Flipped $SU(5)$ Model

There are various symmetry breaking chains in the subsequent  $SO(10) \times U(1)$  breaking. There are two possible symmetry breaking chains for  $E_6$  to

break into flipped SU(5):

$$\begin{aligned} E_6 &\rightarrow SO(10) \times U(1)_1 \rightarrow \text{flipped } SU(5) \times U(1)_1, \\ E_6 &\rightarrow \text{flipped } SO(10) \rightarrow \text{flipped } SU(5). \end{aligned}$$

The  $(\mathbf{45}, \mathbf{0})$  components in  $\mathbf{78}$  and  $\mathbf{650}$  dimensional representation Higgs of  $E_6$  can acquire a VEV which break  $E_6$  into  $SU(5) \times U(1)$ . We will not discuss the subsequent breaking chains of ordinary SO(10) because they have already been discussed in [37]. Here we concentrate on the breaking of flipped SO(10) into flipped SU(5).

The  $\mathbf{78}$  dimensional representation Higgs can acquire Vacuum Expectation Values (VEVs) which break  $E_6$  into  $SU(5) \times U(1)_1 \times U(1)_2$ . Such VEVs can be written as  $27 \times 27$  matrix as follows

$$\langle \Phi \rangle_{(\mathbf{45}, \mathbf{0})}^{\mathbf{78}} = \frac{v_{\mathbf{78}}}{2\sqrt{10}} \text{diag}(\underbrace{-1, \dots, -1}_{10}, \underbrace{3, \dots, 3}_5, -5, \underbrace{2, \dots, 2}_5, \underbrace{-2, \dots, -2}_5, 0), \quad (60)$$

with normalization factor  $c = 3$ . The  $\mathbf{650}$  dimensional Higgs can also acquire Vacuum Expectation Values (VEVs) which break  $E_6$  into  $SU(5) \times U(1)_1 \times U(1)_2$ . Such VEVs can be written as  $27 \times 27$  matrix as follows

$$\langle \Phi \rangle_{(\mathbf{45}, \mathbf{0})}^{\mathbf{650}} = \frac{v_{\mathbf{650}}}{4\sqrt{5}} \text{diag}(\underbrace{1, \dots, 1}_{10}, \underbrace{-3, \dots, -3}_5, 5, \underbrace{4, \dots, 4}_5, \underbrace{-4, \dots, -4}_5, 0), \quad (61)$$

with normalization factor  $c = 3$ .

After  $(\mathbf{45}, \mathbf{0})$  component of  $\mathbf{78}$  dimensional Higgs acquire VEVs, the sfermions acquire masses

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2 - \frac{v_{\mathbf{78}}}{2\sqrt{10}M_*} \beta'^{\mathbf{78}} (m_0^N)^2, \quad (62)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2 - \frac{v_{\mathbf{78}}}{\sqrt{10}M_*} \beta'^{\mathbf{78}} (m_0^N)^2, \quad (63)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2 - \frac{v_{\mathbf{78}}}{2\sqrt{10}M_*} \beta'^{\mathbf{78}} (m_0^N)^2, \quad (64)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2 - \frac{v_{\mathbf{78}}}{\sqrt{10}M_*} \beta'^{\mathbf{78}} (m_0^N)^2, \quad (65)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2. \quad (66)$$

After  $(\mathbf{45}, \mathbf{0})$  component of  $\mathbf{650}$  dimensional Higgs acquire VEVs, the sfermions acquire masses

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2 + \frac{v_{\mathbf{650}}}{4\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (67)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2 - \frac{v_{\mathbf{650}}}{\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (68)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2 + \frac{v_{\mathbf{650}}}{4\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (69)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2 - \frac{v_{\mathbf{650}}}{\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (70)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2. \quad (71)$$

#### 4.3. $E_6$ To $SU(3)_C \times SU(3)_L \times SU(3)_R$ Model

The fundamental representation  $\mathbf{27}$  and adjoint representation  $\mathbf{78}$  of  $E_6$  can be decomposed in term of  $SU(3)_C \times SU(3)_L \times SU(3)_R$

$$\mathbf{27} = (\mathbf{3}, \mathbf{3}, \mathbf{1}) \oplus (\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}) \oplus (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}), \quad (72)$$

$$\mathbf{78} = (\mathbf{8}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{8}) \oplus (\mathbf{3}, \bar{\mathbf{3}}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3}, \mathbf{3}). \quad (73)$$

There are no  $SU(3)_C \times SU(3)_L \times SU(3)_R$  singlet in decomposition of adjoint Higgs  $\Phi(\mathbf{78})$ . So we consider the Vacuum Expectation Values (VEVs) of  $\mathbf{650}$  dimensional representations which can break  $E_6$  into the gauge group  $SU(3)_C \times SU(3)_L \times SU(3)_R$ . There are two singlets in the decomposition of  $\mathbf{650}$  dimensional representations which we can parameter as  $27 \times 27$  matrices.

The two singlets can be recombined to give one left-right symmetric VEVs which preserve the left-right parity and the other left-right non-symmetric VEVs which breaks the left-right parity. The left-right symmetric VEVs can be chosen as

$$\langle \mathbf{650} \rangle_1 = \frac{v_{\mathbf{650}}}{3\sqrt{2}} \text{diag}(\underbrace{-2, \dots, -2}_9, \underbrace{1, \dots, 1}_9, \underbrace{1, \dots, 1}_9), \quad (74)$$

with normalization factor  $c = 3$ . So after  $\mathbf{650}$  dimensional Higgs acquires such VEVs, we can get the soft supersymmetry breaking mass terms for sfermions

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2 + \frac{v_{\mathbf{650}}}{3\sqrt{2}M_*} \beta'_s{}^{\mathbf{650}} (m_0^N)^2, \quad (75)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2 + \frac{v_{\mathbf{650}}}{3\sqrt{2}M_*} \beta_s'^{\mathbf{650}} (m_0^N)^2, \quad (76)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2 + \frac{v_{\mathbf{650}}}{3\sqrt{2}M_*} \beta_s'^{\mathbf{650}} (m_0^N)^2, \quad (77)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2 - 2 \frac{v_{\mathbf{650}}}{3\sqrt{2}M_*} \beta_s'^{\mathbf{650}} (m_0^N)^2, \quad (78)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2 - 2 \frac{v_{\mathbf{650}}}{3\sqrt{2}M_*} \beta_s'^{\mathbf{650}} (m_0^N)^2. \quad (79)$$

The other left-right non-symmetric VEVs can be chosen to be

$$\langle \mathbf{650} \rangle_2 = \frac{\tilde{v}_{\mathbf{650}}}{\sqrt{6}} \text{diag}(\underbrace{0, \dots, 0}_9, \underbrace{1, \dots, 1}_9, \underbrace{-1, \dots, -1}_9), \quad (80)$$

with normalization factor  $c = 3$ . So after **650** dimensional Higgs acquires such VEVs, we can get the soft supersymmetry breaking mass terms for sfermions

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2 + \frac{\tilde{v}_{\mathbf{650}}}{\sqrt{6}M_*} \beta_n'^{\mathbf{650}} (m_0^N)^2, \quad (81)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2 - \frac{\tilde{v}_{\mathbf{650}}}{\sqrt{6}M_*} \beta_n'^{\mathbf{650}} (m_0^N)^2, \quad (82)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2 - \frac{\tilde{v}_{\mathbf{650}}}{\sqrt{6}M_*} \beta_n'^{\mathbf{650}} (m_0^N)^2, \quad (83)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2, \quad (84)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2. \quad (85)$$

#### 4.4. $E_6$ To $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_1 \times U(1)_2$ Model

This symmetry broken chain can be realized via the VEVs of ( **1**, **8**, **8**) components in **650** dimensional representation

$$\langle \mathbf{650} \rangle = \frac{\hat{v}_{\mathbf{650}}}{2\sqrt{3}} \text{diag}(1, 1, -2, 1, 1, -2, -2, -2, 4, \underbrace{0, \dots, 0}_9, \underbrace{0, \dots, 0}_9), \quad (86)$$

with normalization factor  $c = 3$ . So after **650** dimensional Higgs acquires such VEVs, we can get the soft supersymmetry breaking mass terms for

sfermions

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2, \quad (87)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2, \quad (88)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2, \quad (89)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2 - 2 \frac{\hat{v}_{\mathbf{650}}}{2\sqrt{3}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (90)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2 - 2 \frac{\hat{v}_{\mathbf{650}}}{2\sqrt{3}M_*} \beta'^{\mathbf{650}} (m_0^N)^2. \quad (91)$$

Besides, this symmetry broken chain can also be realized by the VEVs of both ( **1**, **1**, **8**) and ( **1**, **8**, **1**) components of **78** dimensional representation

$$< \mathbf{78} >_1 = \frac{v_{\mathbf{78}}}{\sqrt{6}} \text{diag}(\underbrace{0, \dots, 0}_9, \underbrace{1, 1, -2}_3, \underbrace{0, \dots, 0}_9), \quad (92)$$

$$< \mathbf{78} >_2 = \frac{\tilde{v}_{\mathbf{78}}}{\sqrt{6}} \text{diag}(\underbrace{0, \dots, 0}_9, \underbrace{0, \dots, 0}_9, \underbrace{1, 1, -2}_3), \quad (93)$$

with normalization factor  $c = 3$ . This symmetry broken chain can also be realized by the VEVs of both ( **1**, **1**, **8**) and ( **1**, **8**, **1**) components of **650** dimensional representation

$$< \mathbf{650} >_1 = \frac{\hat{v}'_{\mathbf{650}}}{\sqrt{6}} \text{diag}(\underbrace{1, 1, -2}_3, \underbrace{0, \dots, 0}_9, \underbrace{0, \dots, 0}_9), \quad (94)$$

$$< \mathbf{650} >_2 = \frac{\hat{v}''_{\mathbf{650}}}{\sqrt{6}} \text{diag}(\underbrace{1, \dots, 1}_6, \underbrace{-2, \dots, -2}_3, \underbrace{0, \dots, 0}_9, \underbrace{0, \dots, 0}_9), \quad (95)$$

with normalization factor  $c = 3$ . The most general possibilities for  $E_6$  breaking into  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_1 \times U(1)_2$  are realized by both the ( **1**, **1**, **8**) VEVs (from **78** or **650** dimensional representations) and the ( **1**, **8**, **1**) VEVs (from **78** or **650** dimensional representations). Thus the supersymmetry breaking soft mass terms for sfermions

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2 + \frac{v_{\mathbf{78}}}{\sqrt{6}M_*} \beta'^{\mathbf{78}_1} (m_0^N)^2, \quad (96)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2 + \frac{\tilde{v}_{\mathbf{78}}}{\sqrt{6}M_*} \beta'^{\mathbf{78}_2} (m_0^N)^2, \quad (97)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2 + \frac{\tilde{v}_{78}}{\sqrt{6}M_*}\beta'^{782}(m_0^N)^2, \quad (98)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2 + \frac{\hat{v}'_{650}}{\sqrt{6}M_*}\beta'^{6501}(m_0^N)^2 - 2\frac{\hat{v}''_{650}}{\sqrt{6}M_*}\beta'^{6502}(m_0^N)^2, \quad (99)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2 - 2\frac{\hat{v}'_{650}}{\sqrt{6}M_*}\beta'^{6501}(m_0^N)^2 + \frac{\hat{v}''_{650}}{\sqrt{6}M_*}\beta'^{6502}(m_0^N)^2. \quad (100)$$

#### 4.5. $E_6$ To $SU(6) \times SU(2)$ Model

$E_6$  GUT can break into  $SU(6) \times SU(2)$  by **650** dimensional VEVs. According to three different embedding of the standard model matter contents into  $SU(6) \times SU(2)$ , we investigate three different cases according to the three different choices of  $SU(2)$  (namely  $SU(2)_X$ ,  $SU(2)_L$  and  $SU(2)_R$ , respectively). The fundamental representation **27** and adjoint representation **78** of  $E_6$  can be decomposed in term of  $SU(6) \times SU(2)$

$$\mathbf{27} = (\bar{\mathbf{6}}, \mathbf{2}) \oplus (\mathbf{15}, \mathbf{1}), \quad (101)$$

$$\mathbf{78} = (\mathbf{35}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}) \oplus (\mathbf{20}, \mathbf{2}). \quad (102)$$

The VEVs that break  $E_6$  into  $SU(6) \times SU(2)$  can be chosen as

$$\langle \mathbf{650} \rangle = \frac{v_{650}}{6\sqrt{5}} \text{diag}(\underbrace{-4, \dots, -4}_{15}, \underbrace{5, \dots, 5}_{12}), \quad (103)$$

with normalization factor  $c = 3$ .

Then we have three possibilities relating to different filling of the standard model matter contents

- $E_6 \rightarrow SU(6) \times SU(2)_X \rightarrow SU(5) \times U(1) \times SU(2)_X$ :

After **650** dimensional Higgs acquires VEVs, the supersymmetry breaking soft mass terms for sfermions

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2 - \frac{4v_{650}}{6\sqrt{5}M_*}\beta'^{650}(m_0^N)^2, \quad (104)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2 - \frac{4v_{650}}{6\sqrt{5}M_*}\beta'^{650}(m_0^N)^2, \quad (105)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2 + \frac{5v_{650}}{6\sqrt{5}M_*}\beta'^{650}(m_0^N)^2, \quad (106)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2 + \frac{5v_{650}}{6\sqrt{5}M_*}\beta'^{650}(m_0^N)^2, \quad (107)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2 - \frac{4v_{650}}{6\sqrt{5}M_*}\beta'^{650}(m_0^N)^2. \quad (108)$$



- $E_6 \rightarrow SU(6) \times SU(2)_L \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1)_1$ :

After **650** dimensional Higgs acquires VEVs, the supersymmetry broken soft mass terms for sfermions

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2 + \frac{5v_{\mathbf{650}}}{6\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (109)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2 - \frac{4v_{\mathbf{650}}}{6\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (110)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2 - \frac{4v_{\mathbf{650}}}{6\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (111)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2 + \frac{5v_{\mathbf{650}}}{6\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (112)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2 - \frac{4v_{\mathbf{650}}}{6\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2. \quad (113)$$

- $E_6 \rightarrow SU(6) \times SU(2)_R \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1)_2$ :

After **650** dimensional Higgs acquires VEVs, the supersymmetry broken soft mass terms for sfermions

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2 - \frac{4v_{\mathbf{650}}}{6\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (114)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2 + \frac{5v_{\mathbf{650}}}{6\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (115)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2 + \frac{5v_{\mathbf{650}}}{6\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (116)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2 - \frac{4v_{\mathbf{650}}}{6\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (117)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2 + \frac{5v_{\mathbf{650}}}{6\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2. \quad (118)$$

#### 4.6. $E_6$ To $SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1)$ Model

There are two possible symmetry broken chains for  $E_6$  breaking into Pati-Salam model. One symmetry breaking chain is

$$E_6 \rightarrow SO(10) \times U(1) \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1), \quad (119)$$

the other symmetry breaking chain is

$$E_6 \rightarrow SU(6) \times SU(2)_{L,R} \rightarrow SU(4)_c \times SU(2) \times SU(2)_R \times U(1). \quad (120)$$

In this subsection, we concentrate on the second one. Such breaking can be realized via the VEVs of **78** and **650** dimensional representations.

The ( **35**, **1** ) component VEVs of the **78** dimensional representation that break gauge group  $SU(6) \times SU(2)_1$  into  $SU(4) \times SU(2)_1 \times SU(2)_2 \times U(1)$  reads

$$\langle \mathbf{78} \rangle_{(35,1)} = \frac{v_{\mathbf{78}}}{2\sqrt{6}} \text{diag}(\underbrace{1, 1, 1, 1, -2, -2}_2, \underbrace{2, \dots, 2}_6, \underbrace{-1, \dots, -1}_8, -4), \quad (121)$$

with normalization factor  $c = 3$ . The breaking of gauge group  $SU(6) \times SU(2)_1$  into  $SU(4) \times SU(2)_1 \times SU(2)_2 \times U(1)$  can also be realized by both the ( **35**, **1** ) and the ( **189**, **1** ) component VEVs of **650** dimensional representation

$$\begin{aligned} \langle \mathbf{650} \rangle_{(35,1)} &= \frac{v'_{\mathbf{650}}}{2\sqrt{3}} \text{diag}(\underbrace{1, 1, 1, 1, -2, -2}_2, \underbrace{-1, \dots, -1}_6, \underbrace{\frac{1}{2}, \dots, \frac{1}{2}}_8, 2), \\ \langle \mathbf{650} \rangle_{(189,1)} &= \frac{\tilde{v}'_{\mathbf{650}}}{4\sqrt{5}} \text{diag}(\underbrace{0, \dots, 0}_{12}, \underbrace{-2, \dots, -2}_6, \underbrace{3, \dots, 3}_8, -12), \end{aligned} \quad (122)$$

with normalization factor  $c = 3$ .

- $E_6 \rightarrow SU(6) \times SU(2)_L \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1)_1$ :

After the ( **35**, **1** ) component of **78** dimensional Higgs acquires VEVs, we can get the soft supersymmetry breaking mass terms for sfermions

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2 + \frac{v_{\mathbf{78}}}{2\sqrt{6}M_*} \beta'^{\mathbf{78}} (m_0^N)^2, \quad (123)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2 - \frac{v_{\mathbf{78}}}{2\sqrt{6}M_*} \beta'^{\mathbf{78}} (m_0^N)^2, \quad (124)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2 - \frac{v_{\mathbf{78}}}{2\sqrt{6}M_*} \beta'^{\mathbf{78}} (m_0^N)^2, \quad (125)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2 + \frac{v_{\mathbf{78}}}{2\sqrt{6}M_*} \beta'^{\mathbf{78}} (m_0^N)^2, \quad (126)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2 - \frac{v_{\mathbf{78}}}{2\sqrt{6}M_*} \beta'^{\mathbf{78}} (m_0^N)^2. \quad (127)$$

After the ( **35**, **1**) component of **650** dimensional Higgs acquires VEVs, we can get the soft supersymmetry breaking mass terms for sfermions

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2 + \frac{v'_{\mathbf{650}}}{2\sqrt{3}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (128)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2 + \frac{v'_{\mathbf{650}}}{4\sqrt{3}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (129)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2 + \frac{v'_{\mathbf{650}}}{4\sqrt{3}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (130)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2 + \frac{v'_{\mathbf{650}}}{2\sqrt{3}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (131)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2 + \frac{v'_{\mathbf{650}}}{4\sqrt{3}M_*} \beta'^{\mathbf{650}} (m_0^N)^2. \quad (132)$$

After the ( **189**, **1**) component of **650** dimensional Higgs acquires VEVs, we can get the soft supersymmetry breaking mass terms for sfermions

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2, \quad (133)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2 + \frac{3\tilde{v}'_{\mathbf{650}}}{4\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (134)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2 + \frac{3\tilde{v}'_{\mathbf{650}}}{4\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (135)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2, \quad (136)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2 + \frac{3\tilde{v}'_{\mathbf{650}}}{4\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2. \quad (137)$$

- $E_6 \rightarrow SU(6) \times SU(2)_R \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1)_2$ :

After the ( **35**, **1**) component of **78** dimensional Higgs acquires VEVs, we can get the soft supersymmetry breaking mass terms for sfermions

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2 - \frac{v_{\mathbf{78}}}{2\sqrt{6}M_*} \beta'^{\mathbf{78}} (m_0^N)^2, \quad (138)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2 + \frac{v_{\mathbf{78}}}{2\sqrt{6}M_*} \beta'^{\mathbf{78}} (m_0^N)^2, \quad (139)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2 + \frac{v_{\mathbf{78}}}{2\sqrt{6}M_*} \beta'^{\mathbf{78}} (m_0^N)^2, \quad (140)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2 - \frac{v_{\mathbf{78}}}{2\sqrt{6}M_*} \beta'^{\mathbf{78}} (m_0^N)^2, \quad (141)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2 + \frac{v_{\mathbf{78}}}{2\sqrt{6}M_*} \beta'^{\mathbf{78}} (m_0^N)^2. \quad (142)$$

After the ( **35**, **1** ) component of **650** dimensional Higgs acquires VEVs, we can get the soft supersymmetry breaking mass terms for sfermions

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2 + \frac{v'_{\mathbf{650}}}{4\sqrt{3}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (143)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2 + \frac{v'_{\mathbf{650}}}{2\sqrt{3}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (144)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2 + \frac{v'_{\mathbf{650}}}{2\sqrt{3}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (145)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2 + \frac{v'_{\mathbf{650}}}{4\sqrt{3}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (146)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2 + \frac{v'_{\mathbf{650}}}{2\sqrt{3}M_*} \beta'^{\mathbf{650}} (m_0^N)^2. \quad (147)$$

After the ( **189**, **1** ) component of **650** dimensional Higgs acquires VEVs, we can get the soft supersymmetry breaking mass terms for sfermions

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2 + \frac{3\tilde{v}'_{\mathbf{650}}}{4\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (148)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2, \quad (149)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2, \quad (150)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2 + \frac{3\tilde{v}'_{\mathbf{650}}}{4\sqrt{5}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (151)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2, \quad (152)$$

#### 4.7. $E_6$ To $SU(5) \times U(1) \times SU(2)_X$ Model

The breaking of  $E_6$  into  $SU(5) \times U(1) \times SU(2)_X$  can be realized via the VEVs of **78** and **650** dimensional representations.

The ( **35**, **1** ) component VEVs of the **78** dimensional representation that break  $SU(6) \times SU(2)_X$  to  $SU(5) \times U(1) \times SU(2)_X$  reads

$$\langle \mathbf{78} \rangle_{(\mathbf{35}, \mathbf{1})} = \frac{\hat{v}_{\mathbf{78}}}{2\sqrt{15}} \text{diag}(\underbrace{1, 1, 1, 1, 1}_{2}, \underbrace{2, \dots, 2}_{10}, \underbrace{-4, \dots, -4}_{5}), \quad (153)$$

with normalization factor  $c = 3$ . The ( **35**, **1** ) component VEVs of the **650** dimensional representation that break gauge group  $SU(6) \times SU(2)_X$  into group  $SU(5) \times U(1) \times SU(2)_X$  reads

$$\langle \mathbf{650} \rangle_{(\mathbf{35}, \mathbf{1})} = \frac{\hat{v}_{\mathbf{650}}}{\sqrt{30}} \text{diag}(\underbrace{1, 1, 1, 1, 1, -5}_2, \underbrace{-1, \dots, -1}_{10}, \underbrace{2, \dots, 2}_5), \quad (154)$$

with normalization factor  $c = 3$ . After the ( **35**, **1** ) component of **78** dimensional Higgs acquires VEVs, we can get the soft supersymmetry breaking mass terms for sfermions

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2 + \frac{\hat{v}_{\mathbf{78}}}{\sqrt{15}M_*} \beta'^{\mathbf{78}} (m_0^N)^2, \quad (155)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2 + \frac{\hat{v}_{\mathbf{78}}}{\sqrt{15}M_*} \beta'^{\mathbf{78}} (m_0^N)^2, \quad (156)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2 + \frac{\hat{v}_{\mathbf{78}}}{2\sqrt{15}M_*} \beta'^{\mathbf{78}} (m_0^N)^2, \quad (157)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2 + \frac{\hat{v}_{\mathbf{78}}}{2\sqrt{15}M_*} \beta'^{\mathbf{78}} (m_0^N)^2, \quad (158)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2 + \frac{\hat{v}_{\mathbf{78}}}{\sqrt{15}M_*} \beta'^{\mathbf{78}} (m_0^N)^2. \quad (159)$$

After the ( **35**, **1** ) component of **650** dimensional Higgs acquires VEVs, we can get the soft supersymmetry breaking mass terms for sfermions

$$m_{\tilde{Q}_L}^2 = (m_0^U)^2 - \frac{\hat{v}_{\mathbf{650}}}{\sqrt{30}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (160)$$

$$m_{\tilde{U}_L^C}^2 = (m_0^U)^2 - \frac{\hat{v}_{\mathbf{650}}}{\sqrt{30}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (161)$$

$$m_{\tilde{D}_L^C}^2 = (m_0^U)^2 + \frac{\hat{v}_{\mathbf{650}}}{\sqrt{30}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (162)$$

$$m_{\tilde{L}_L}^2 = (m_0^U)^2 + \frac{\hat{v}_{\mathbf{650}}}{\sqrt{30}M_*} \beta'^{\mathbf{650}} (m_0^N)^2, \quad (163)$$

$$m_{\tilde{E}_L^C}^2 = (m_0^U)^2 - \frac{\hat{v}_{\mathbf{650}}}{\sqrt{30}M_*} \beta'^{\mathbf{650}} (m_0^N)^2. \quad (164)$$

## 5. MSSM Superpotential and Soft Trilinear Terms in $E_6$ SUSY GUT

To get new contributions to MSSM superpotential and soft trilinear terms from higher-dimensional operators, we need to consider the group tensor production decomposition for the Yukawa coupling [65]

$$\begin{aligned}
\mathbf{27}_m \otimes \mathbf{27}_m \otimes \mathbf{27}_H &= (\overline{\mathbf{27}}_s \oplus \overline{\mathbf{351}}_a \oplus \overline{\mathbf{351}}'_s) \otimes \mathbf{27} \\
&= (\mathbf{1} \oplus \mathbf{78} \oplus \mathbf{650}) \oplus (\mathbf{78} \oplus \mathbf{650} \oplus \mathbf{2925} \oplus \overline{\mathbf{5824}}) \\
&\oplus (\mathbf{650} \oplus \mathbf{3003} \oplus \overline{\mathbf{5824}}) .
\end{aligned} \tag{165}$$

We consider in this paper the effect of  $\mathbf{78}$ ,  $\mathbf{650}$  dimensional representation Higgs to superpotential and trilinear terms. For  $\mathbf{78}$  dimensional representation Higgs fields, we consider the following non-renormalizable superpotential

$$\begin{aligned}
W \supset & \frac{1}{M_*} \left[ h_{ij}^a [(\mathbf{27}_m^i \otimes \mathbf{27}_m^j)_{\overline{\mathbf{351}}}^a \otimes \mathbf{78}] \otimes \mathbf{27}_H \right. \\
& \left. + (h'_{ij})^s (\mathbf{27}_m^i \otimes \mathbf{27}_m^j)_{\overline{\mathbf{27}}}^s \otimes (\mathbf{27}_H \otimes \mathbf{78}) \right] \\
& + \alpha \frac{T}{M_*^2} \left[ y_{ij}^a [(\mathbf{27}_m^i \otimes \mathbf{27}_m^j)_{\overline{\mathbf{351}}}^a \otimes \mathbf{78}] \otimes \mathbf{27}_H \right. \\
& \left. + (y'_{ij})^s (\mathbf{27}_m^i \otimes \mathbf{27}_m^j)_{\overline{\mathbf{27}}}^s \otimes (\mathbf{27}_H \otimes \mathbf{78}) \right] .
\end{aligned} \tag{166}$$

The two terms corresponds to two linearly independent contraction methods in the group production. The superscripts ' $s$ ' ( or ' $a$ ') indicates that the coefficients are symmetric (or antisymmetric) with respect to the family ' $ij$ ' index.

For  $\mathbf{650}$  dimensional representation Higgs fields, we consider the following non-renormalizable superpotential

$$\begin{aligned}
W \supset & \frac{1}{M_*} \left[ h_{ij}^a [(\mathbf{27}_m^i \otimes \mathbf{27}_m^j)_{\overline{\mathbf{351}}}^a \otimes \mathbf{650}] \otimes \mathbf{27}_H \right. \\
& \left. + h'^s_{ij} [(\mathbf{27}_m^i \otimes \mathbf{27}_m^j)_{\overline{\mathbf{351}}}^s \otimes \mathbf{650}] \otimes \mathbf{27}_H + (h''_{ij})^s (\mathbf{27}_m^i \otimes \mathbf{27}_m^j)_{\overline{\mathbf{27}}}^s \otimes (\mathbf{27}_H \otimes \mathbf{650}) \right] \\
& + \alpha \frac{T}{M_*^2} \left[ y_{ij}^a [(\mathbf{27}_m^i \otimes \mathbf{27}_m^j)_{\overline{\mathbf{351}}}^a \otimes \mathbf{650}] \otimes \mathbf{27}_H \right. \\
& \left. + y'^s_{ij} [(\mathbf{27}_m^i \otimes \mathbf{27}_m^j)_{\overline{\mathbf{351}}}^s \otimes \mathbf{650}] \otimes \mathbf{27}_H + (y''_{ij})^s (\mathbf{27}_m^i \otimes \mathbf{27}_m^j)_{\overline{\mathbf{27}}}^s \otimes (\mathbf{27}_H \otimes \mathbf{650}) \right] .
\end{aligned} \tag{167}$$

The three terms corresponds to three linearly independent contraction methods in the group production. The superscripts ' $s$ ' ( or ' $a$ ') indicates that the coefficients are symmetric (or antisymmetric) with respect to the family ' $ij$ ' index.

### 5.1. $E_6$ To $SO(10) \times U(1)$ Model

The **78** dimensional representation Higgs can acquire Vacuum Expectation Values (VEVs) which break  $E_6$  into  $SO(10) \times U(1)$ . Such VEVs can be written as  $27 \times 27$  matrix as follows

$$\langle \Phi \rangle^{\mathbf{78}} = \frac{\hat{v}_{\mathbf{78}}}{2\sqrt{6}} \text{diag}(\underbrace{1, \dots, 1}_{16}, \underbrace{-2, \dots, -2}_{10}, 4) , \quad (168)$$

with normalization factor  $c = 3$ . The **650** dimensional Higgs can also acquire Vacuum Expectation Values which break  $E_6$  into  $SO(10) \times U(1)$ . Such VEVs can be written as  $27 \times 27$  matrix as follows

$$\langle \Phi \rangle^{\mathbf{650}} = \frac{\hat{v}_{\mathbf{650}}}{12\sqrt{5}} \text{diag}(\underbrace{-5, \dots, -5}_{16}, \underbrace{4, \dots, 4}_{10}, 40) , \quad (169)$$

with normalization factor  $c = 3$ .

- U(1) Extension of Ordinary SO(10):

The gauge invariant Yukawa coupling in  $E_6$  GUT have the form

$$\begin{aligned} W &\supset \sum_{i,j=1}^3 y_{ij} \mathbf{27}^i \mathbf{27}^j \mathbf{27}_h \supset \sum_{i,j=1}^3 y_{ij} \mathbf{16}^i \mathbf{16}^j \mathbf{10}_H \\ &\supset \sum_{i,j=1}^3 2y_{ij}^s [Q_L^i (U_L^c)^j h_u + Q_L^i (D_L^c)^j h_d + L_L^i (E_L^c)^j h_d + L_L^i (N_L^c)^j h_u] . \end{aligned} \quad (170)$$

After the  $(\mathbf{45}, \mathbf{1})$  component of **78** dimensional Higgs acquire VEVs which is denoted by  $\langle \mathbf{78} \rangle_{(\mathbf{45}, \mathbf{1})}$ , the new contributions to superpotential

$$\begin{aligned} W &\supset \frac{\hat{v}_{\mathbf{78}}}{2\sqrt{6}M_*} \sum_{i,j=1}^3 \left[ h_{ij}^a \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H - 2h_{ij}^{'s} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H \right] , \\ &\supset \frac{\hat{v}_{\mathbf{78}}}{2\sqrt{6}M_*} \sum_{i,j=1}^3 \left[ -2h_{ij}^{'s} \{ 2Q_L^i (U_L^c)^j H_u + 2Q_L^i (D_L^c)^j H_d \right. \\ &\quad \left. + 2L_L^i (E_L^c)^j H_d + 2L_L^i (N_L^c)^j H_u \} \right] , \end{aligned} \quad (171)$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$\begin{aligned}
-\mathcal{L} \supset & \alpha' \frac{\hat{v}_{78} F_T}{2\sqrt{6} M_*^2} \sum_{i,j=1}^3 \left[ -2y'_{ij} \{ 2\tilde{Q}_L^i (\tilde{U}_L^c)^j H_u + 2\tilde{Q}_L^i (\tilde{D}_L^c)^j H_d \right. \\
& \left. + 2\tilde{L}_L^i (\tilde{E}_L^c)^j H_d + 2\tilde{L}_L^i (\tilde{N}_L^c)^j H_u \} \right]. \quad (172)
\end{aligned}$$

After (45, 1) component of **650** dimensional Higgs acquire VEVs which is denoted by  $\langle \mathbf{650} \rangle_{(45,1)}$ , the new contributions to superpotential

$$\begin{aligned}
W \supset & \frac{\hat{v}_{650}}{12\sqrt{5} M_*} \sum_{i,j=1}^3 \left[ -5h_{ij}^a \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H - 5h_{ij}'^s \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H \right. \\
& \left. + 4h_{ij}''^s \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H \right], \\
\supset & \frac{\hat{v}_{650}}{12\sqrt{5} M_*} \sum_{i,j=1}^3 \left[ 2(-5h_{ij}'^s + 4h_{ij}''^s) \{ Q_L^i (U_L^c)^j H_u + Q_L^i (D_L^c)^j H_d \right. \\
& \left. + L_L^i (E_L^c)^j H_d + L_L^i (N_L^c)^j H_u \} \right], \quad (173)
\end{aligned}$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$\begin{aligned}
-\mathcal{L} \supset & \alpha' \frac{\hat{v}_{650} F_T}{2\sqrt{6} M_*^2} \sum_{i,j=1}^3 \left[ 2(-5y_{ij}'^s + 4y_{ij}''^s) \{ \tilde{Q}_L^i (\tilde{U}_L^c)^j H_u + \tilde{Q}_L^i (\tilde{D}_L^c)^j H_d \right. \\
& \left. + \tilde{L}_L^i (\tilde{E}_L^c)^j H_d + \tilde{L}_L^i (\tilde{N}_L^c)^j H_u \} \right]. \quad (174)
\end{aligned}$$

- Flipped SO(10):

The gauge invariant Yukawa coupling in  $E_6$  GUT have the form

$$\begin{aligned}
W \supset & \sum_{i,j=1}^3 y_{ij} \mathbf{27}^i \mathbf{27}^j \mathbf{27}_h \\
\supset & \sum_{i,j=1}^3 y_{ij} \mathbf{16}_m^i \mathbf{16}_m^j \mathbf{10}_H + 2y_{ij}^s \mathbf{16}_m^i \mathbf{10}_m^j \mathbf{16}_H + 2y_{ij}^s \mathbf{10}_m \mathbf{1}_m \mathbf{10}_H
\end{aligned}$$



$$\supset \sum_{i,j=1}^3 2y_{ij}^s [Q_L^i (U_L^c)^j h_u + Q_L^i (D_L^c)^j h_d + L_L^i (E_L^c)^j h_d + L_L^i (N_L^c)^j h_u]. \quad (175)$$

After the ( **45**, **1**) component of **78** dimensional Higgs acquire VEVs which is denoted by  $\langle \mathbf{78} \rangle_{(45,1)}$ , the new contributions to superpotential

$$\begin{aligned} W &\supset \frac{\hat{v}_{78}}{2\sqrt{6}M_*} \sum_{i,j=1}^3 \left[ h_{ij}^a \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H + 3h_{ij}^a \mathbf{16}_m^i \mathbf{10}_m^j \mathbf{16}_H \right. \\ &\quad \left. - 6h_{ij}^a \mathbf{10}_m \mathbf{1}_m \mathbf{10}_H - 2h_{ij}^{'s} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H + 2h_{ij}^{'s} \mathbf{16}_m^i \mathbf{10}_m^j \mathbf{16}_H \right. \\ &\quad \left. - 4h_{ij}^{'s} \mathbf{10}_m \mathbf{1}_m \mathbf{10}_H \right], \\ &\supset \frac{\hat{v}_{78}}{2\sqrt{6}M_*} \sum_{i,j=1}^3 \left[ -4h_{ij}^{'s} Q_L^i (D_L^c)^j H_d + (3h_{ij}^a + 2h_{ij}^{'s}) \{Q_L^i (U_L^c)^j H_u \right. \\ &\quad \left. + L_L^i (E_L^c)^j H_d\} + (-6h_{ij}^a - 4h_{ij}^{'s}) L_L^i (N_L^c)^j H_u \right], \end{aligned} \quad (176)$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$\begin{aligned} -\mathcal{L} &\supset \alpha' \frac{\hat{v}_{78} F_T}{2\sqrt{6}M_*^2} \sum_{i,j=1}^3 \left[ -4y_{ij}^{'s} \tilde{Q}_L^i (\tilde{D}_L^c)^j H_d + (3y_{ij}^a + 2y_{ij}^{'s}) \{ \tilde{Q}_L^i (\tilde{U}_L^c)^j H_u \right. \\ &\quad \left. + \tilde{L}_L^i (\tilde{E}_L^c)^j H_d\} + (-6y_{ij}^a - 4y_{ij}^{'s}) \tilde{L}_L^i (\tilde{N}_L^c)^j H_u \right]. \end{aligned} \quad (177)$$

After ( **45**, **1**) component of **650** dimensional Higgs acquire VEVs which is denoted by  $\langle \mathbf{650} \rangle_{(45,1)}$ , the new contributions to superpotential

$$\begin{aligned} W &\supset \frac{\hat{v}_{650}}{12\sqrt{5}M_*} \sum_{i,j=1}^3 \left[ 44h_{ij}^{'s} \mathbf{10}_m \mathbf{1}_m \mathbf{10}_H - 5h_{ij}^a \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H \right. \\ &\quad - 9h_{ij}^a \mathbf{16}_m^i \mathbf{10}_m^j \mathbf{16}_H - 5h_{ij}^{'s} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H - h_{ij}^{'s} \mathbf{16}_m^i \mathbf{10}_m^j \mathbf{16}_H \\ &\quad - 36h_{ij}^a \mathbf{10}_m \mathbf{1}_m \mathbf{10}_H + 4h_{ij}^{'s} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H - 10h_{ij}^{'s} \mathbf{16}_m^i \mathbf{10}_m^j \mathbf{16}_H \\ &\quad \left. + 8h_{ij}^{'s} \mathbf{10}_m \mathbf{1}_m \mathbf{10}_H \right], \end{aligned}$$

$$\begin{aligned}
\supset \frac{\hat{v}_{\mathbf{650}}}{12\sqrt{5}M_*} \sum_{i,j=1}^3 & \left[ (-10h'_{ij} + 8h''_{ij}) Q_L^i (D_L^c)^j H_d \right. \\
& + (-9h_{ij}^a - h'_{ij}{}^s - 10h''_{ij}{}^s) \{ Q_L^i (U_L^c)^j H_u + L_L^i (E_L^c)^j H_d \} \\
& \left. + (-36h_{ij}^a + 44h'_{ij}{}^s + 8h''_{ij}{}^s) L_L^i (N_L^c)^j H_u \right], \quad (178)
\end{aligned}$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$\begin{aligned}
-\mathcal{L} \supset \alpha' \frac{v_{\mathbf{650}} F_T}{12\sqrt{5}M_*^2} \sum_{i,j=1}^3 & \left[ (-10y'_{ij} + 8y''_{ij}) \tilde{Q}_L^i (\tilde{D}_L^c)^j H_d \right. \\
& + (-9y_{ij}^a - y'_{ij}{}^s - 10y''_{ij}{}^s) \{ \tilde{Q}_L^i (\tilde{U}_L^c)^j H_u + \tilde{L}_L^i (\tilde{E}_L^c)^j H_d \} \\
& \left. + (-36y_{ij}^a + 44y'_{ij}{}^s + 8y''_{ij}{}^s) \tilde{L}_L^i (\tilde{N}_L^c)^j H_u \right]. \quad (179)
\end{aligned}$$

### 5.2. $E_6$ To Flipped $SU(5)$ Model

As before, we will not discuss new contributions to the trilinear terms from subsequent breaking chains of ordinary  $SO(10)$  because they have already been discussed in our previous works [37]. Here we concentrate on the breaking of flipped  $SO(10)$  into flipped  $SU(5)$ .

The **78** dimensional representation Higgs can acquire Vacuum Expectation Values which break  $E_6$  into  $SU(5) \times U(1)_1 \times U(1)_2$ . Such VEVs can be written as  $27 \times 27$  matrix as follows

$$\langle \Phi \rangle_{(\mathbf{45},0)}^{\mathbf{78}} = \frac{v_{\mathbf{78}}}{2\sqrt{10}} \text{diag}(\underbrace{-1, \dots, -1}_{10}, \underbrace{3, \dots, 3}_5, -5, \underbrace{2, \dots, 2}_5, \underbrace{-2, \dots, -2}_5, 0), \quad (180)$$

with normalization factor  $c = 3$ . The **650** dimensional Higgs can also acquire Vacuum Expectation Values which break  $E_6$  gauge group into its subgroup  $SU(5) \times U(1)_1 \times U(1)_2$ . Such VEVs can be written as  $27 \times 27$  matrix as follows

$$\langle \Phi \rangle_{(\mathbf{45},0)}^{\mathbf{650}} = \frac{v_{\mathbf{650}}}{4\sqrt{5}} \text{diag}(\underbrace{1, \dots, 1}_{10}, \underbrace{-3, \dots, -3}_5, 5, \underbrace{4, \dots, 4}_5, \underbrace{-4, \dots, -4}_5, 0), \quad (181)$$

with normalization factor  $c = 3$ .

After the ( **45**, **1** ) component of **78** dimensional Higgs acquire VEVs which is denoted by  $\langle \mathbf{78} \rangle_{(\mathbf{45}, \mathbf{1})}$ , the new contributions to superpotential

$$\begin{aligned}
W &\supset \frac{v_{\mathbf{78}}}{2\sqrt{10}M_*} \sum_{i,j=1}^3 \left[ -h_{ij}^a \mathbf{10}_i \mathbf{10}_j \mathbf{5}_H + h_{ij}^a \mathbf{10}_m^i \bar{\mathbf{5}}_m^j \bar{\mathbf{5}}_H - 2h_{ij}^a \bar{\mathbf{5}}_m \mathbf{1}_m \mathbf{5}_H \right. \\
&\quad \left. + 2h_{ij}'^s \mathbf{10}_i \mathbf{10}_j \mathbf{5}_H - 4h_{ij}'^s \mathbf{10}_m^i \bar{\mathbf{5}}_m^j \bar{\mathbf{5}}_H + 4h_{ij}'^s \bar{\mathbf{5}}_m \mathbf{1}_m \mathbf{5}_H \right], \\
&\supset \frac{v_{\mathbf{78}}}{2\sqrt{10}M_*} \sum_{i,j=1}^3 \left[ 4h_{ij}'^s Q_L^i (D_L^c)^j H_d + (-2h_{ij}^a + 4h_{ij}'^s) L_L^i (N_L^c)^j H_u \right. \\
&\quad \left. + (h_{ij}^a - 4h_{ij}'^s) \{ Q_L^i (U_L^c)^j H_u + L_L^i (E_L^c)^j H_d \} \right], \quad (182)
\end{aligned}$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$\begin{aligned}
-\mathcal{L} &\supset \alpha' \frac{v_{\mathbf{78}} F_T}{2\sqrt{10}M_*^2} \sum_{i,j=1}^3 \left[ 4y_{ij}'^s \tilde{Q}_L^i (\tilde{D}_L^c)^j H_d + (-2y_{ij}^a + 4y_{ij}'^s) \tilde{L}_L^i (\tilde{N}_L^c)^j H_u \right. \\
&\quad \left. + (y_{ij}^a - 4y_{ij}'^s) \{ \tilde{Q}_L^i (\tilde{U}_L^c)^j H_u + \tilde{L}_L^i (\tilde{E}_L^c)^j H_d \} \right]. \quad (183)
\end{aligned}$$

After ( **45**, **1** ) component of **650** dimensional Higgs acquire VEVs which is denoted by  $\langle \mathbf{650} \rangle_{(\mathbf{45}, \mathbf{1})}$ , the new contributions to superpotential

$$\begin{aligned}
W &\supset \frac{v_{\mathbf{650}}}{4\sqrt{5}M_*} \sum_{i,j=1}^3 \left[ h_{ij}^a \mathbf{10}_i \mathbf{10}_j \mathbf{5}_H + 5h_{ij}^a \mathbf{10}_m^i \bar{\mathbf{5}}_m^j \bar{\mathbf{5}}_H - 4h_{ij}^a \bar{\mathbf{5}}_m \mathbf{1}_m \mathbf{5}_H \right. \\
&\quad + h_{ij}'^s \mathbf{10}_i \mathbf{10}_j \mathbf{5}_H - 3h_{ij}'^s \mathbf{10}_m^i \bar{\mathbf{5}}_m^j \bar{\mathbf{5}}_H - 4h_{ij}'^s \bar{\mathbf{5}}_m \mathbf{1}_m \mathbf{5}_H \\
&\quad \left. + 4h_{ij}''^s \mathbf{10}_i \mathbf{10}_j \mathbf{5}_H - 8h_{ij}''^s \mathbf{10}_m^i \bar{\mathbf{5}}_m^j \bar{\mathbf{5}}_H + 8h_{ij}''^s \bar{\mathbf{5}}_m \mathbf{1}_m \mathbf{5}_H \right], \\
&\supset \frac{v_{\mathbf{650}}}{4\sqrt{5}M_*} \sum_{i,j}^3 \left[ (h_{ij}'^s + 4h_{ij}''^s) Q_L^i (D_L^c)^j H_d \right. \\
&\quad + (-4h_{ij}^a - 4h_{ij}'^s + 8h_{ij}''^s) L_L^i (N_L^c)^j H_u \\
&\quad \left. + (5h_{ij}^a - 3h_{ij}'^s - 8h_{ij}''^s) \{ Q_L^i (U_L^c)^j H_u + L_L^i (E_L^c)^j H_d \} \right], \quad (184)
\end{aligned}$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$-\mathcal{L} \supset \alpha' \frac{v_{\mathbf{650}} F_T}{4\sqrt{5}M_*^2} \sum_{i,j=1}^3 \left[ (y_{ij}'^s + 4y_{ij}''^s) \tilde{Q}_L^i (\tilde{D}_L^c)^j H_d \right.$$

$$\begin{aligned}
& - (4y_{ij}^a + 4y_{ij}'^s - 8y_{ij}''^s) \tilde{L}_L^i (\tilde{N}_L^c)^j H_u \} \\
& + (5y_{ij}^a - 3y_{ij}'^s - 8y_{ij}''^s) \{ \tilde{Q}_L^i (\tilde{U}_L^c)^j H_u + \tilde{L}_L^i (\tilde{E}_L^c)^j H_d \} \Big] . \quad (185)
\end{aligned}$$

### 5.3. $E_6$ To $SU(3)_C \times SU(3)_L \times SU(3)_R$ Model

The gauge invariant Yukawa coupling in  $E_6$  GUT have the form

$$\begin{aligned}
W & \supset \sum_{i,j=1}^3 y_{ij} \mathbf{27}^i \mathbf{27}^j \mathbf{27}_h \supset \sum_{i,j}^3 (2y_{ij}^s X_L^i (X_L^c)^j H + y_{ij} N^i N^j H) \\
& \supset \sum_{i,j=1}^3 y_{ij}^s \left[ 2Q_L^i (Q_L^c)^j \Phi + 2L_L^i (L_L^c)^j \Phi \right] \\
& \supset \sum_{i,j=1}^3 y_{ij}^s \left[ 2Q_L^i (U_L^c)^j h_u + 2Q_L^i (D_L^c)^j h_d + 2L_L^i (E_L^c)^j h_d + 2L_L^i (N_L^c)^j h_u \right] .
\end{aligned} \quad (186)$$

in which we identify the  $(\mathbf{1}_c, \bar{\mathbf{3}}, \mathbf{3})$  components of the Higgs fields  $\mathbf{27}_H$  as  $H$ ; the bi-doublets  $(\mathbf{1}_c, \mathbf{2}, \mathbf{2})$  in  $H$  as  $\Phi$ ;  $y_{ij}$  is decomposed into symmetric  $y_{ij}^s$  and antisymmetric  $y_{ij}^a$  parts.

We know that the breaking of  $E_6$  into  $SU(3)_C \times SU(3)_L \times SU(3)_R$  are realized by VEVs of **650** dimensional representation Higgs fields. As noted before, the left-right symmetric VEVs can be chosen as

$$\langle \mathbf{650} \rangle_1 = \frac{v_{\mathbf{650}}}{3\sqrt{2}} \text{diag}(\underbrace{-2, \dots, -2}_9, \underbrace{1, \dots, 1}_9, \underbrace{1, \dots, 1}_9) , \quad (187)$$

while the left-right non-symmetric VEVs can be chosen as

$$\langle \mathbf{650} \rangle_2 = \frac{\tilde{v}_{\mathbf{650}}}{\sqrt{6}} \text{diag}(\underbrace{0, \dots, 0}_9, \underbrace{1, \dots, 1}_9, \underbrace{-1, \dots, -1}_9) , \quad (188)$$

with normalization factor  $c = 3$ .

After **650** dimensional Higgs acquire left-right symmetric VEVs  $\langle \mathbf{650} \rangle_1$ , the new contributions to superpotential

$$W \supset \frac{v_{\mathbf{650}}}{3\sqrt{2}M_*} \sum_{i,j=1}^3 \left\{ h_{ij}'^s [2X_L^i (X_L^c)^j H - 2N^i N^j H] \right.$$

$$\begin{aligned}
& - 2h_{ij}''^s [2X_L^i (X_L^c)^j H + N^i N^j H] \Big\}, \\
\supset & \frac{v_{\mathbf{650}}}{3\sqrt{2}M_*} \sum_{i,j=1}^3 \Big\{ h_{ij}'^s [2Q_L^i (Q_L^c)^j \Phi - 4L_L^i (L_L^c)^j \Phi] \\
& - 4h_{ij}''^s [Q_L^i (Q_L^c)^j \Phi + L_L^i (L_L^c)^j \Phi] \Big\}, \\
\supset & \frac{v_{\mathbf{650}}}{3\sqrt{2}M_*} \sum_{i,j=1}^3 \Big[ (2h_{ij}'^s - 4h_{ij}''^s) \{2Q_L^i (U_L^c)^j h_u + 2Q_L^i (D_L^c)^j h_d\} \\
& - 4(h_{ij}'^s + h_{ij}''^s) \{L_L^i (E_L^c)^j h_d + L_L^i (N_L^c)^j h_u\} \Big], \quad (189)
\end{aligned}$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$\begin{aligned}
-\mathcal{L} \supset & \alpha' \frac{v_{\mathbf{650}} F_T}{3\sqrt{2}M_*^2} \sum_{i,j=1}^3 \Big\{ (2y_{ij}'^s - 4y_{ij}''^s) [\tilde{Q}_L^i (\tilde{U}_L^c)^j h_u + \tilde{Q}_L^i (\tilde{D}_L^c)^j h_d] \\
& - 4(y_{ij}'^s + y_{ij}''^s) [\tilde{L}_L^i (\tilde{E}_L^c)^j h_d + \tilde{L}_L^i (\tilde{N}_L^c)^j h_u] \Big\}. \quad (190)
\end{aligned}$$

The **650** dimensional Higgs can also acquire left-right non-symmetric VEVs  $\langle \mathbf{650} \rangle_2$ , so the new contribution to superpotential

$$\begin{aligned}
W \supset & \frac{\tilde{v}_{\mathbf{650}}}{\sqrt{6}M_*} \sum_{i,j=1}^3 \Big\{ 2h_{ij}^a X_L^i (X_L^c)^j H \Big\}, \\
\supset & \frac{\tilde{v}_{\mathbf{650}}}{\sqrt{6}M_*} \sum_{i,j=1}^3 \Big[ h_{ij}^a \{2Q_L^i (U_L^c)^j h_u + 2Q_L^i (D_L^c)^j h_d\} \Big], \quad (191)
\end{aligned}$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$-\mathcal{L} \supset \alpha' \frac{\tilde{v}_{\mathbf{650}} F_T}{\sqrt{6}M_*^2} \sum_{i,j=1}^3 \Big\{ 2y_{ij}^a \tilde{Q}_L^i (\tilde{U}_L^c)^j h_u + 2y_{ij}^a \tilde{Q}_L^i (\tilde{D}_L^c)^j h_d \Big\}. \quad (192)$$

#### 5.4. $E_6$ To $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_1 \times U(1)_2$ Model

We know that this symmetry broken chain can be realized by the VEVs of ( **1**, **8**, **8**) components in **650** dimensional representation Higgs fields

$$\langle \mathbf{650} \rangle = \frac{\hat{v}_{\mathbf{650}}}{2\sqrt{3}} \text{diag}(1, 1, -2, 1, 1, -2, -2, -2, 4, \underbrace{0, \dots, 0}_9, \underbrace{0, \dots, 0}_9). \quad (193)$$

with normalization  $c = 3$ . After **650** dimensional Higgs acquire left-right symmetric VEVs  $\langle \mathbf{650} \rangle$ , the new contributions to superpotential

$$\begin{aligned}
W &\supset \frac{\hat{v}_{\mathbf{650}}}{2\sqrt{3}M_*} \sum_{i,j=1}^3 \left\{ -4h'_{ij} L_L^i (L_L^c)^j \Phi + h''_{ij} [2Q_L^i (Q_L^c)^j \Phi + 2L_L^i (L_L^c)^j \Phi] \right\}, \\
&\supset \frac{\hat{v}_{\mathbf{650}}}{2\sqrt{3}M_*} \sum_{i,j=1}^3 \left[ (-4h'_{ij} + 2h''_{ij}) \{ L_L^i (E_L^c)^j h_d + L_L^i (N_L^c)^j h_u \} \right. \\
&\quad \left. + 2h''_{ij} \{ Q_L^i (U_L^c)^j h_u + Q_L^i (D_L^c)^j h_d \} \right]. \quad (194)
\end{aligned}$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$\begin{aligned}
-\mathcal{L} &\supset \alpha' \frac{\hat{v}_{\mathbf{650}} F_T}{2\sqrt{3}M_*^2} \sum_{i,j=1}^3 \left\{ (-4y'_{ij} + 2y''_{ij}) \left[ \tilde{L}_L^i (\tilde{E}_L^c)^j h_d + \tilde{L}_L^i (\tilde{N}_L^c)^j h_u \right] \right. \\
&\quad \left. + 2y''_{ij} \left[ \tilde{Q}_L^i (\tilde{U}_L^c)^j h_u + \tilde{Q}_L^i (\tilde{D}_L^c)^j h_d \right] \right\}. \quad (195)
\end{aligned}$$

As noted before, this symmetry broken chain can also be realized by the VEVs of both  $(\mathbf{1}, \mathbf{1}, \mathbf{8})$  and  $(\mathbf{1}, \mathbf{8}, \mathbf{1})$  components of **78** dimensional representation Higgs fields

$$\langle \mathbf{78} \rangle_1 = \frac{v_{\mathbf{78}}}{\sqrt{6}} \text{diag}(\underbrace{-1, -1, 2}_3, \underbrace{1, 1, -2}_3, \underbrace{0, \dots, 0}_9), \quad (196)$$

$$\langle \mathbf{78} \rangle_2 = \frac{\tilde{v}_{\mathbf{78}}}{\sqrt{6}} \text{diag}(\underbrace{-1, \dots, -1}_6, \underbrace{2, \dots, 2}_3, \underbrace{0, \dots, 0}_9, \underbrace{1, 1, -2}_3), \quad (197)$$

with  $c = 3$ .

Besides, it is also possible for this symmetry broken chain to be realized by the VEVs of both  $(\mathbf{1}, \mathbf{1}, \mathbf{8})$  and  $(\mathbf{1}, \mathbf{8}, \mathbf{1})$  components of **650** dimensional representation Higgs fields

$$\langle \mathbf{650} \rangle_1 = \frac{\hat{v}'_{\mathbf{650}}}{\sqrt{6}} \text{diag}(\underbrace{1, 1, -2}_3, \underbrace{1, 1, -2}_3, \underbrace{0, \dots, 0}_9), \quad (198)$$

$$\langle \mathbf{650} \rangle_2 = \frac{\hat{v}''_{\mathbf{650}}}{\sqrt{6}} \text{diag}(\underbrace{1, \dots, 1}_6, \underbrace{-2, \dots, -2}_3, \underbrace{0, \dots, 0}_9, \underbrace{1, 1, -2}_3), \quad (199)$$

with  $c = 3$ .

After both ( **1**, **1**, **8**) and ( **1**, **8**, **1**) components in **78** dimensional Higgs acquire VEVs, the new contributions to superpotential

$$\begin{aligned}
W &\supset \frac{v_{\mathbf{78}}}{\sqrt{6}M_*} \left\{ h_{1ij}^a Q_L^i (Q_L^c)^j \Phi - 3h_{1ij}^a L_L^i (L_L^c)^j \Phi + h_{1ij}'^s Q_L^i (Q_L^c)^j \Phi \right. \\
&\quad \left. + h_{1ij}'^s L_L^i (L_L^c)^j \Phi - 2h_{1ij}''^s Q_L^i (Q_L^c)^j \Phi - 2h_{1ij}''^s L_L^i (L_L^c)^j \Phi \right\}, \\
&+ \frac{\tilde{v}_{\mathbf{78}}}{\sqrt{6}M_*} \left\{ -h_{2ij}^a Q_L^i (Q_L^c)^j \Phi + 3h_{2ij}^a L_L^i (L_L^c)^j \Phi + h_{2ij}'^s Q_L^i (Q_L^c)^j \Phi \right. \\
&\quad \left. + h_{2ij}'^s L_L^i (L_L^c)^j \Phi - 2h_{2ij}''^s Q_L^i (Q_L^c)^j \Phi - 2h_{2ij}''^s L_L^i (L_L^c)^j \Phi \right\}, \\
&\supset \frac{v_{\mathbf{78}}}{\sqrt{6}M_*} \left[ (h_{1ij}^a + h_{1ij}'^s - 2h_{1ij}''^s) \{ Q_L^i (U_L^c)^j h_u + Q_L^i (D_L^c)^j h_d \} \right. \\
&\quad \left. + (-3h_{1ij}^a + h_{1ij}'^s - 2h_{1ij}''^s) \{ L_L^i (E_L^c)^j h_d + L_L^i (N_L^c)^j h_u \} \right] \\
&+ \frac{\tilde{v}_{\mathbf{78}}}{\sqrt{6}M_*} \left[ (-h_{2ij}^a + h_{2ij}'^s - 2h_{2ij}''^s) \{ Q_L^i (U_L^c)^j h_u + Q_L^i (D_L^c)^j h_d \} \right. \\
&\quad \left. + (3h_{2ij}^a + h_{2ij}'^s - 2h_{2ij}''^s) \{ L_L^i (E_L^c)^j h_d + L_L^i (N_L^c)^j h_u \} \right]. \quad (200)
\end{aligned}$$

while the supersymmetry breaking soft trilinear terms

$$\begin{aligned}
-\mathcal{L} &\supset \alpha' \frac{v_{\mathbf{78}} F_T}{\sqrt{6}M_*^2} \left[ (y_{1ij}^a + y_{1ij}'^s - 2y_{1ij}''^s) \{ \tilde{Q}_L^i (\tilde{U}_L^c)^j h_u + \tilde{Q}_L^i (\tilde{D}_L^c)^j h_d \} \right. \\
&\quad \left. + (-3y_{1ij}^a + y_{1ij}'^s - 2y_{1ij}''^s) \{ \tilde{L}_L^i (\tilde{E}_L^c)^j h_d + \tilde{L}_L^i (\tilde{N}_L^c)^j h_u \} \right] \\
&+ \alpha' \frac{\tilde{v}_{\mathbf{78}} F_T}{\sqrt{6}M_*^2} \left[ (-y_{2ij}^a + y_{2ij}'^s - 2y_{2ij}''^s) \{ \tilde{Q}_L^i (\tilde{U}_L^c)^j h_u + \tilde{Q}_L^i (\tilde{D}_L^c)^j h_d \} \right. \\
&\quad \left. + (3y_{2ij}^a + y_{2ij}'^s - 2y_{2ij}''^s) \{ \tilde{L}_L^i (\tilde{E}_L^c)^j h_d + \tilde{L}_L^i (\tilde{N}_L^c)^j h_u \} \right]. \quad (201)
\end{aligned}$$

Similarly, after both ( **1**, **1**, **8**) and ( **1**, **8**, **1**) components in **650** dimensional Higgs acquire VEVs, the new contributions to superpotential

$$\begin{aligned}
W &\supset \frac{v_{\mathbf{650}}}{\sqrt{6}M_*} \left\{ h_{1ij}^a Q_L^i (Q_L^c)^j \Phi + 3h_{1ij}^a L_L^i (L_L^c)^j \Phi + h_{1ij}'^s Q_L^i (Q_L^c)^j \Phi \right. \\
&\quad \left. - h_{1ij}'^s L_L^i (L_L^c)^j \Phi + 2h_{1ij}''^s Q_L^i (Q_L^c)^j \Phi + 2h_{1ij}''^s L_L^i (L_L^c)^j \Phi \right\}, \\
&+ \frac{\tilde{v}_{\mathbf{650}}}{\sqrt{6}M_*} \left\{ -h_{2ij}^a Q_L^i (Q_L^c)^j \Phi - 3h_{2ij}^a L_L^i (L_L^c)^j \Phi + h_{2ij}'^s Q_L^i (Q_L^c)^j \Phi \right. \\
&\quad \left. - h_{2ij}'^s L_L^i (L_L^c)^j \Phi + 2h_{2ij}''^s Q_L^i (Q_L^c)^j \Phi + 2h_{2ij}''^s L_L^i (L_L^c)^j \Phi \right\},
\end{aligned}$$

$$\begin{aligned}
\supset & \frac{v_{\mathbf{650}}}{\sqrt{6}M_*} \left[ (h_{1ij}^a + h_{1ij}'^s + 2h_{1ij}''^s) \{Q_L^i(U_L^c)^j h_u + Q_L^i(D_L^c)^j h_d\} \right. \\
& \quad \left. + (3h_{1ij}^a - h_{1ij}'^s + 2h_{1ij}''^s) \{L_L^i(E_L^c)^j h_d + L_L^i(N_L^c)^j h_u\} \right] \\
& + \frac{\tilde{v}_{\mathbf{650}}}{\sqrt{6}M_*} \left[ (-h_{2ij}^a + h_{2ij}'^s + 2h_{2ij}''^s) \{Q_L^i(U_L^c)^j h_u + Q_L^i(D_L^c)^j h_d\} \right. \\
& \quad \left. + (-3h_{2ij}^a - h_{2ij}'^s + 2h_{2ij}''^s) \{L_L^i(E_L^c)^j h_d + L_L^i(N_L^c)^j h_u\} \right] , \quad (202)
\end{aligned}$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$\begin{aligned}
-\mathcal{L} \supset & \alpha' \frac{v_{\mathbf{78}} F_T}{\sqrt{6}M_*^2} \left[ (y_{1ij}^a + y_{1ij}'^s + 2y_{1ij}''^s) \{\tilde{Q}_L^i(\tilde{U}_L^c)^j h_u + \tilde{Q}_L^i(\tilde{D}_L^c)^j h_d\} \right. \\
& \quad \left. + (3y_{1ij}^a - y_{1ij}'^s + 2y_{1ij}''^s) \{\tilde{L}_L^i(\tilde{E}_L^c)^j h_d + \tilde{L}_L^i(\tilde{N}_L^c)^j h_u\} \right] \\
& + \alpha' \frac{\tilde{v}_{\mathbf{78}} F_T}{\sqrt{6}M_*^2} \left[ (-y_{2ij}^a + y_{2ij}'^s + 2y_{2ij}''^s) \{\tilde{Q}_L^i(\tilde{U}_L^c)^j h_u + \tilde{Q}_L^i(\tilde{D}_L^c)^j h_d\} \right. \\
& \quad \left. + (-3y_{2ij}^a - y_{2ij}'^s + 2y_{2ij}''^s) \{\tilde{L}_L^i(\tilde{E}_L^c)^j h_d + \tilde{L}_L^i(\tilde{N}_L^c)^j h_u\} \right] . \quad (203)
\end{aligned}$$

### 5.5. $E_6$ To $SU(6) \times SU(2)$ Model

We know that this symmetry broken chain can be realized via the VEVs of  $\mathbf{650}$  dimensional representation Higgs field. The VEVs that break  $E_6$  into  $SU(6) \times SU(2)$  can be chosen as

$$\langle \mathbf{650} \rangle = \frac{v_{\mathbf{650}}}{6\sqrt{5}} \text{diag}(\underbrace{-4, \dots, -4}_{15}, \underbrace{5, \dots, 5}_{12}) , \quad (204)$$

with normalization factor  $c = 3$ .

- $E_6 \rightarrow SU(6) \times SU(2)_X \rightarrow SU(5) \times U(1) \times SU(2)_X$ : The filling of matter contents can be seen in previous sections. The gauge invariant (renormalizable) Yukawa coupling in  $E_6$  GUT thus have the form

$$\begin{aligned}
W & \supset \sum_{i,j=1}^3 y_{ij} \mathbf{27}^i \mathbf{27}^j \mathbf{27}_h , \\
& \supset \sum_{i,j=1}^3 \left( y_{ij}^s F_{(\mathbf{10},1)}^i F_{(\mathbf{10},1)}^j H_{(\mathbf{5},1)} + 2y_{ij}^s F_{(\mathbf{10},1)}^i F_{(\mathbf{5},2)}^j H_{(\mathbf{5},2)} \right)
\end{aligned}$$



$$\begin{aligned}
& + 2y_{ij}^s F_{(\bar{\mathbf{5}},\mathbf{2})}^i F_{(\mathbf{1},\mathbf{2})}^j H_{(\mathbf{5},\mathbf{1})} \Big) , \\
\supset & \sum_{i,j=1}^3 y_{ij}^s \left[ 2Q_L^i (U_L^c)^j h_u + 2Q_L^i (D_L^c)^j h_d + 2L_L^i (E_L^c)^j h_d \right. \\
& \left. + 2L_L^i (N_L^c)^j h_u \right] . \tag{205}
\end{aligned}$$

After **650** dimensional Higgs acquire VEVs  $\langle \mathbf{650} \rangle$ , the new contributions to superpotential

$$\begin{aligned}
W \supset & \frac{v_{\mathbf{650}}}{6\sqrt{5}M_*} \sum_{i,j=1}^3 \left( -9h_{ij}^a F_{(\mathbf{10},\mathbf{1})}^i F_{(\bar{\mathbf{5}},\mathbf{2})}^j H_{(\mathbf{5},\mathbf{2})} \right) , \\
& + \frac{v_{\mathbf{650}}}{6\sqrt{5}M_*} \sum_{i,j=1}^3 \left( -4h_{ij}'^s F_{(\mathbf{10},\mathbf{1})}^i F_{(\mathbf{10},\mathbf{1})}^j H_{(\mathbf{5},\mathbf{1})} + h_{ij}'^s F_{(\mathbf{10},\mathbf{1})}^i F_{(\bar{\mathbf{5}},\mathbf{2})}^j H_{(\mathbf{5},\mathbf{2})} \right. \\
& \quad \left. + 10h_{ij}''^s F_{(\bar{\mathbf{5}},\mathbf{2})}^i F_{(\mathbf{1},\mathbf{2})}^j H_{(\mathbf{5},\mathbf{1})} \right) , \\
& + \frac{v_{\mathbf{650}}}{6\sqrt{5}M_*} \sum_{i,j=1}^3 \left( -4h_{ij}''^s F_{(\mathbf{10},\mathbf{1})}^i F_{(\mathbf{10},\mathbf{1})}^j H_{(\mathbf{5},\mathbf{1})} + 10h_{ij}''^s F_{(\mathbf{10},\mathbf{1})}^i F_{(\bar{\mathbf{5}},\mathbf{2})}^j H_{(\mathbf{5},\mathbf{2})} \right. \\
& \quad \left. - 8h_{ij}''^s F_{(\bar{\mathbf{5}},\mathbf{2})}^i F_{(\mathbf{1},\mathbf{2})}^j H_{(\mathbf{5},\mathbf{1})} \right) , \\
\supset & \frac{v_{\mathbf{650}}}{6\sqrt{5}M_*} \sum_{i,j=1}^3 \left[ -8(h_{ij}'^s + h_{ij}''^s) Q_L^i (U_L^c)^j h_u \right. \\
& \quad \left. + (-9h_{ij}^a + h_{ij}'^s + 10h_{ij}''^s) Q_L^i (D_L^c)^j h_d \right] , \\
& + \frac{v_{\mathbf{650}}}{6\sqrt{5}M_*} \sum_{i,j=1}^3 \left[ (10h_{ij}'^s - 8h_{ij}''^s) L_L^i (N_L^c)^j h_u \right. \\
& \quad \left. + (-9h_{ij}^a + h_{ij}'^s + 10h_{ij}''^s) L_L^i (E_L^c)^j h_d \right] , \tag{206}
\end{aligned}$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$\begin{aligned}
-\mathcal{L} \supset & \alpha' \frac{v_{\mathbf{650}} F_T}{6\sqrt{5}M_*^2} \sum_{i,j=1}^3 \left[ -8(y_{ij}'^s + y_{ij}''^s) \tilde{Q}_L^i (\tilde{U}_L^c)^j h_u \right. \\
& \quad \left. + (-9y_{ij}^a + y_{ij}'^s + 10y_{ij}''^s) \tilde{Q}_L^i (\tilde{D}_L^c)^j h_d \right] ,
\end{aligned}$$

$$\begin{aligned}
& + \alpha' \frac{v_{\mathbf{650}} F_T}{6\sqrt{5}M_*^2} \sum_{i,j=1}^3 \left[ (10y'_{ij}{}^s - 8y''_{ij}{}^s) \tilde{L}_L^i (\tilde{N}_L^c)^j h_u \right. \\
& \quad \left. + (-9y_{ij}^a + y'_{ij}{}^s + 10y''_{ij}{}^s) \tilde{L}_L^i (\tilde{E}_L^c)^j h_d \right]. \quad (207)
\end{aligned}$$

- $E_6 \rightarrow SU(6) \times SU(2)_L \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1)_1$ : The gauge invariant Yukawa coupling in  $E_6$  GUT thus have the form

$$\begin{aligned}
W & \supset \sum_{i,j=1}^3 y_{ij} \mathbf{27}^i \mathbf{27}^j \mathbf{27}_h, \\
& \supset \sum_{i,j=1}^3 \left( 2y_{ij}^s F_{(\mathbf{4},\mathbf{2},\mathbf{1})}^i F_{(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})}^j H_{(\mathbf{1},\bar{\mathbf{2}},\mathbf{2})} \right), \\
& \supset \sum_{i,j=1}^3 y_{ij}^s \left[ 2Q_R^i (Q_R^c)^j \Phi + 2L_R^i (L_R^c)^j \Phi \right], \\
& \supset - \sum_{i,j=1}^3 y_{ij}^s \{ 2Q_L^i (U_L^c)^j H_u + 2Q_L^i (D_L^c)^j H_d \\
& \quad + 2L_L^i (E_L^c)^j H_d + 2L_L^i (N_L^c)^j H_u \}. \quad (208)
\end{aligned}$$

After **650** dimensional Higgs acquire VEVs  $\langle \mathbf{650} \rangle_1$ , the new contributions to superpotential

$$\begin{aligned}
W & \supset \frac{v_{\mathbf{650}}}{6\sqrt{5}M_*} \sum_{i,j}^3 \left[ -9h_{ij}^a F_{(\mathbf{4},\mathbf{2},\mathbf{1})}^i F_{(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})}^j H_{(\mathbf{1},\bar{\mathbf{2}},\mathbf{2})} \right. \\
& \quad \left. + h_{ij}^s F_{(\mathbf{4},\mathbf{2},\mathbf{1})}^i F_{(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})}^j H_{(\mathbf{1},\bar{\mathbf{2}},\mathbf{2})} + 10h_{ij}''^s F_{(\mathbf{4},\mathbf{2},\mathbf{1})}^i F_{(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})}^j H_{(\mathbf{1},\bar{\mathbf{2}},\mathbf{2})} \right], \\
& \supset \frac{v_{\mathbf{650}}}{6\sqrt{5}M_*} \sum_{i,j}^3 \left[ (9h_{ij}^a + h_{ij}^s + 10h_{ij}''^s) \left\{ 2Q_L^i (U_L^c)^j H_u \right. \right. \\
& \quad \left. \left. + 2Q_L^i (D_L^c)^j H_d + 2L_L^i (E_L^c)^j H_d + 2L_L^i (N_L^c)^j H_u \right\} \right], \quad (209)
\end{aligned}$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$-\mathcal{L} \supset \alpha' \frac{v_{\mathbf{650}} F_T}{6\sqrt{5}M_*^2} \sum_{i,j}^3 \left[ (9y_{ij}^a + y_{ij}^s + 10y_{ij}''^s) \left\{ 2\tilde{Q}_L^i (\tilde{U}_L^c)^j H_u \right. \right.$$

$$+2\tilde{Q}_L^i(\tilde{D}_L^c)^j H_d + 2\tilde{L}_L^i(\tilde{E}_L^c)^j H_d + 2\tilde{L}_L^i(\tilde{N}_L^c)^j H_u \} \Big] .(210)$$

- $E_6 \rightarrow SU(6) \times SU(2)_R \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1)_1$ :

The gauge invariant Yukawa coupling in  $E_6$  GUT thus have the form

$$\begin{aligned} W &\supset \sum_{i,j=1}^3 y_{ij} \mathbf{27}^i \mathbf{27}^j \mathbf{27}_h , \\ &\supset \sum_{i,j=1}^3 \left( 2y_{ij}^s F_{(\mathbf{4},\mathbf{2},\mathbf{1})}^i F_{(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})}^j H_{(\mathbf{1},\bar{\mathbf{2}},\mathbf{2})} \right) , \\ &\supset \sum_{i,j=1}^3 y_{ij}^s \left[ 2Q_L^i (Q_L^c)^j \Phi + 2L_L^i (L_L^c)^j \Phi \right] , \\ &\supset \sum_{i,j=1}^3 y_{ij}^s \{ 2Q_L^i (U_L^c)^j H_u + 2Q_L^i (D_L^c)^j H_d \\ &\quad + 2L_L^i (E_L^c)^j H_d + 2L_L^i (N_L^c)^j H_u \} . \end{aligned} \quad (211)$$

After **650** dimensional Higgs acquire VEVs  $\langle \mathbf{650} \rangle$ , the new contributions to superpotential

$$\begin{aligned} W &\supset \frac{v_{\mathbf{650}}}{6\sqrt{5}M_*} \sum_{i,j=1}^3 \left[ -9h_{ij}^a F_{(\mathbf{4},\mathbf{2},\mathbf{1})}^i F_{(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})}^j H_{(\mathbf{1},\bar{\mathbf{2}},\mathbf{2})} \right. \\ &\quad \left. + h_{ij}^{'s} F_{(\mathbf{4},\mathbf{2},\mathbf{1})}^i F_{(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})}^j H_{(\mathbf{1},\bar{\mathbf{2}},\mathbf{2})} + 10h_{ij}^{''s} F_{(\mathbf{4},\mathbf{2},\mathbf{1})}^i F_{(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})}^j H_{(\mathbf{1},\bar{\mathbf{2}},\mathbf{2})} \right] , \\ &\supset \frac{v_{\mathbf{650}}}{6\sqrt{5}M_*} \sum_{i,j=1}^3 \left[ (-9h_{ij}^a + h_{ij}^{'s} + 10h_{ij}^{''s}) \left\{ 2Q_L^i (U_L^c)^j H_u \right. \right. \\ &\quad \left. \left. + 2Q_L^i (D_L^c)^j H_d + 2L_L^i (E_L^c)^j H_d + 2L_L^i (N_L^c)^j H_u \right\} \right] .(212) \end{aligned}$$

while the supersymmetry breaking soft trilinear terms

$$\begin{aligned} -\mathcal{L} &\supset \alpha' \frac{v_{\mathbf{650}} F_T}{6\sqrt{5}M_*^2} \sum_{i,j=1}^3 \left[ (-9y_{ij}^a + y_{ij}^{'s} + 10y_{ij}^{''s}) \left\{ 2\tilde{Q}_L^i (\tilde{U}_L^c)^j H_u \right. \right. \\ &\quad \left. \left. + 2\tilde{Q}_L^i (\tilde{D}_L^c)^j H_d + 2\tilde{L}_L^i (\tilde{E}_L^c)^j H_d + 2\tilde{L}_L^i (\tilde{N}_L^c)^j H_u \right\} \right] .(213) \end{aligned}$$

### 5.6. $E_6$ To $SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1)$ Model

We know that this symmetry broken chain can be realized via the VEVs of **650** as well as **78** dimensional representation Higgs field. The  $(\mathbf{35}, \mathbf{1})$  component VEVs of the **78** dimensional representation that break gauge group  $SU(6) \times SU(2)_1$  to its subgroup  $SU(4) \times SU(2)_1 \times SU(2)_2 \times U(1)$  reads

$$\langle \mathbf{78} \rangle_{(\mathbf{35}, \mathbf{1})} = \frac{v_{\mathbf{78}}}{2\sqrt{6}} \text{diag}(\underbrace{1, 1, 1, 1, -2, -2}_2, \underbrace{2, \dots, 2}_6, \underbrace{-1, \dots, -1}_8, -4), \quad (214)$$

with normalization factor  $c = 3$ . The breaking of gauge group  $SU(6) \times SU(2)_1$  to its subgroup  $SU(4) \times SU(2)_1 \times SU(2)_2 \times U(1)$  can be realized by both the  $(\mathbf{35}, \mathbf{1})$  and the  $(\mathbf{189}, \mathbf{1})$  component VEVs of **650** dimensional representation

$$\begin{aligned} \langle \mathbf{650} \rangle_{(\mathbf{35}, \mathbf{1})} &= \frac{v'_{\mathbf{650}}}{2\sqrt{3}} \text{diag}(\underbrace{1, 1, 1, 1, -2, -2}_2, \underbrace{-1, \dots, -1}_6, \underbrace{\frac{1}{2}, \dots, \frac{1}{2}}_8, 2), \\ \langle \mathbf{650} \rangle_{(\mathbf{189}, \mathbf{1})} &= \frac{\tilde{v}'_{\mathbf{650}}}{4\sqrt{5}} \text{diag}(\underbrace{0, \dots, 0}_{12}, \underbrace{-2, \dots, -2}_6, \underbrace{3, \dots, 3}_8, -12), \end{aligned} \quad (215)$$

with normalization factor  $c = 3$ .

- $E_6 \rightarrow SU(6) \times SU(2)_L \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1)_1$ :

After the  $(\mathbf{35}, \mathbf{1})$  component of **78** dimensional Higgs acquire VEVs  $\langle \mathbf{78} \rangle_{(\mathbf{35}, \mathbf{1})}$ , the new contributions to superpotential

$$\begin{aligned} W &\supset \frac{v_{\mathbf{78}}}{2\sqrt{6}M_*} \sum_{i,j=1}^3 \left[ -4h'_{ij} F_{(\mathbf{4}, \mathbf{2}, \mathbf{1})}^i F_{(\mathbf{4}, \mathbf{1}, \mathbf{2})}^j H_{(\mathbf{1}, \mathbf{2}, \mathbf{2})} \right], \\ &\supset \frac{v_{\mathbf{78}}}{2\sqrt{6}M_*} \sum_{i,j=1}^3 \left[ -4h'_{ij} \{ 2Q_L^i (U_L^c)^j H_u \right. \\ &\quad \left. + 2Q_L^i (D_L^c)^j H_d + 2L_L^i (E_L^c)^j H_d + 2L_L^i (N_L^c)^j H_u \} \right], \end{aligned} \quad (216)$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$-\mathcal{L} \supset \alpha' \frac{v_{\mathbf{78}} F_T}{2\sqrt{6}M_*^2} \sum_{i,j=1}^3 \left[ -4y'_{ij} \{ 2\tilde{Q}_L^i (\tilde{U}_L^c)^j H_u \right.$$

$$+2\tilde{Q}_L^i(\tilde{D}_L^c)^j H_d + 2\tilde{L}_L^i(\tilde{E}_L^c)^j H_d + 2\tilde{L}_L^i(\tilde{N}_L^c)^j H_u \} \Big] . \quad (217)$$

After **(35, 1)** component of **650** dimensional Higgs acquire VEVs which is denoted by  $\langle \mathbf{650} \rangle_{(35,1)}$ , the new contributions to superpotential

$$\begin{aligned} W &\supset \frac{v'_{\mathbf{650}}}{2\sqrt{3}M_*} \sum_{i,j=1}^3 \left[ -\frac{3}{2} h_{ij}^a F_{(\mathbf{4},\mathbf{2},\mathbf{1})}^i F_{(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})}^j H_{(\mathbf{1},\bar{\mathbf{2}},\mathbf{2})} \right. \\ &\quad \left. + \frac{1}{2} h_{ij}'^s F_{(\mathbf{4},\mathbf{2},\mathbf{1})}^i F_{(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})}^j H_{(\mathbf{1},\bar{\mathbf{2}},\mathbf{2})} - 4h_{ij}''^s F_{(\mathbf{4},\mathbf{2},\mathbf{1})}^i F_{(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})}^j H_{(\mathbf{1},\bar{\mathbf{2}},\mathbf{2})} \right] , \\ &\supset \frac{v'_{\mathbf{650}}}{2\sqrt{3}M_*} \sum_{i,j=1}^3 \left[ \left( \frac{3}{2} h_{ij}^a + \frac{1}{2} h_{ij}'^s - 4h_{ij}''^s \right) \left\{ 2Q_L^i (U_L^c)^j H_u \right. \right. \\ &\quad \left. \left. + 2Q_L^i (D_L^c)^j H_d + 2L_L^i (E_L^c)^j H_d + 2L_L^i (N_L^c)^j H_u \right\} \right] , \quad (218) \end{aligned}$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$\begin{aligned} -\mathcal{L} &\supset \alpha' \frac{v'_{\mathbf{650}} F_T}{2\sqrt{3}M_*^2} \sum_{i,j=1}^3 \left[ \left( \frac{3}{2} y_{ij}^a + \frac{1}{2} y_{ij}'^s - 4y_{ij}''^s \right) \left\{ 2\tilde{Q}_L^i (\tilde{U}_L^c)^j H_u \right. \right. \\ &\quad \left. \left. + 2\tilde{Q}_L^i (\tilde{D}_L^c)^j H_d + 2\tilde{L}_L^i (\tilde{E}_L^c)^j H_d + 2\tilde{L}_L^i (\tilde{N}_L^c)^j H_u \right\} \right] . \quad (219) \end{aligned}$$

After **(189, 1)** component of **650** dimensional Higgs acquire VEVs which is denoted by  $\langle \mathbf{650} \rangle_{(189,1)}$ , the new contributions to superpotential

$$\begin{aligned} W &\supset \frac{\tilde{v}'_{\mathbf{650}}}{6\sqrt{5}M_*} \sum_{i,j=1}^3 \left[ -3h_{ij}^a F_{(\mathbf{4},\mathbf{2},\mathbf{1})}^i F_{(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})}^j H_{(\mathbf{1},\bar{\mathbf{2}},\mathbf{2})} \right. \\ &\quad \left. - 3h_{ij}'^s F_{(\mathbf{4},\mathbf{2},\mathbf{1})}^i F_{(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})}^j H_{(\mathbf{1},\bar{\mathbf{2}},\mathbf{2})} \right] , \\ &\supset \frac{\tilde{v}'_{\mathbf{650}}}{6\sqrt{5}M_*} \sum_{i,j=1}^3 \left[ (3h_{ij}^a - 3h_{ij}'^s) \left\{ 2Q_L^i (U_L^c)^j H_u \right. \right. \\ &\quad \left. \left. + 2Q_L^i (D_L^c)^j H_d + 2L_L^i (E_L^c)^j H_d + 2L_L^i (N_L^c)^j H_u \right\} \right] , \quad (220) \end{aligned}$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$\begin{aligned}
-\mathcal{L} \supset & \alpha' \frac{\tilde{v}'_{\mathbf{650}} F_T}{6\sqrt{5}M_*^2} \sum_{i,j=1}^3 \left[ (3y_{ij}^a - 3y_{ij}'^s) \left\{ 2\tilde{Q}_L^i (\tilde{U}_L^c)^j H_u \right. \right. \\
& \left. \left. + 2\tilde{Q}_L^i (\tilde{D}_L^c)^j H_d + 2\tilde{L}_L^i (\tilde{E}_L^c)^j H_d + 2\tilde{L}_L^i (\tilde{N}_L^c)^j H_u \right\} \right]. \quad (221)
\end{aligned}$$

- $E_6 \rightarrow SU(6) \times SU(2)_R \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1)_1$ :

After the ( **35**, **1** ) component of **78** dimensional Higgs acquire VEVs which is denoted by  $\langle \mathbf{78} \rangle_{(\mathbf{35}, \mathbf{1})}$ , the new contributions to superpotential

$$\begin{aligned}
W & \supset \frac{v_{\mathbf{78}}}{2\sqrt{6}M_*} \sum_{i,j=1}^3 \left[ -4h_{ij}'^s F_{(\mathbf{4}, \mathbf{2}, \mathbf{1})}^i F_{(\mathbf{4}, \mathbf{1}, \mathbf{2})}^j H_{(\mathbf{1}, \mathbf{2}, \mathbf{2})} \right], \\
& \supset \frac{v_{\mathbf{78}}}{2\sqrt{6}M_*} \sum_{i,j=1}^3 \left[ -4h_{ij}'^s \{ 2Q_L^i (U_L^c)^j H_u \right. \\
& \left. + 2Q_L^i (D_L^c)^j H_d + 2L_L^i (E_L^c)^j H_d + 2L_L^i (N_L^c)^j H_u \} \right], \quad (222)
\end{aligned}$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$\begin{aligned}
-\mathcal{L} \supset & \alpha \frac{v_{\mathbf{78}} F_T}{2\sqrt{6}M_*^2} \sum_{i,j=1}^3 \left[ -4y_{ij}'^s \{ 2\tilde{Q}_L^i (\tilde{U}_L^c)^j H_u \right. \\
& \left. + 2\tilde{Q}_L^i (\tilde{D}_L^c)^j H_d + 2\tilde{L}_L^i (\tilde{E}_L^c)^j H_d + 2\tilde{L}_L^i (\tilde{N}_L^c)^j H_u \} \right]. \quad (223)
\end{aligned}$$

After ( **35**, **1** ) component of **650** dimensional Higgs acquire VEVs which is denoted by  $\langle \mathbf{650} \rangle_{(\mathbf{35}, \mathbf{1})}$ , the new contributions to superpotential

$$\begin{aligned}
W & \supset \frac{v'_{\mathbf{650}}}{2\sqrt{3}M_*} \sum_{i,j=1}^3 \left[ -\frac{3}{2} h_{ij}^a F_{(\mathbf{4}, \mathbf{2}, \mathbf{1})}^i F_{(\mathbf{4}, \mathbf{1}, \mathbf{2})}^j H_{(\mathbf{1}, \mathbf{2}, \mathbf{2})} \right. \\
& \left. + \frac{1}{2} h_{ij}'^s F_{(\mathbf{4}, \mathbf{2}, \mathbf{1})}^i F_{(\mathbf{4}, \mathbf{1}, \mathbf{2})}^j H_{(\mathbf{1}, \mathbf{2}, \mathbf{2})} - 4h_{ij}''^s F_{(\mathbf{4}, \mathbf{2}, \mathbf{1})}^i F_{(\mathbf{4}, \mathbf{1}, \mathbf{2})}^j H_{(\mathbf{1}, \mathbf{2}, \mathbf{2})} \right], \\
& \supset \frac{v'_{\mathbf{650}}}{2\sqrt{3}M_*} \sum_{i,j=1}^3 \left[ \left( -\frac{3}{2} h_{ij}^a + \frac{1}{2} h_{ij}'^s - 4h_{ij}''^s \right) \left\{ 2Q_L^i (U_L^c)^j H_u \right. \right. \\
& \left. \left. + 2Q_L^i (D_L^c)^j H_d + 2L_L^i (E_L^c)^j H_d + 2L_L^i (N_L^c)^j H_u \right\} \right], \quad (224)
\end{aligned}$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$\begin{aligned}
-\mathcal{L} \supset & \alpha' \frac{v'_{\mathbf{650}} F_T}{2\sqrt{3}M_*^2} \sum_{i,j=1}^3 \left[ \left( -\frac{3}{2}y_{ij}^a + \frac{1}{2}y_{ij}'^s - 4y_{ij}''^s \right) \left\{ 2\tilde{Q}_L^i (\tilde{U}_L^c)^j H_u \right. \right. \\
& \left. \left. + 2\tilde{Q}_L^i (\tilde{D}_L^c)^j H_d + 2\tilde{L}_L^i (\tilde{E}_L^c)^j H_d + 2\tilde{L}_L^i (\tilde{N}_L^c)^j H_u \right\} \right]. \quad (225)
\end{aligned}$$

After  $(\mathbf{189}, \mathbf{1})$  component of  $\mathbf{650}$  dimensional Higgs acquire VEVs which is denoted by  $\langle \mathbf{650} \rangle_{(\mathbf{189}, \mathbf{1})}$ , the new contributions to superpotential

$$\begin{aligned}
W \supset & \frac{\tilde{v}'_{\mathbf{650}}}{6\sqrt{5}M_*} \sum_{i,j=1}^3 \left[ -3h_{ij}^a F_{(\mathbf{4}, \mathbf{2}, \mathbf{1})}^i F_{(\mathbf{4}, \mathbf{1}, \mathbf{2})}^j H_{(\mathbf{1}, \mathbf{2}, \mathbf{2})} \right. \\
& \left. - 3h_{ij}'^s F_{(\mathbf{4}, \mathbf{2}, \mathbf{1})}^i F_{(\mathbf{4}, \mathbf{1}, \mathbf{2})}^j H_{(\mathbf{1}, \mathbf{2}, \mathbf{2})} \right], \\
\supset & \frac{\tilde{v}'_{\mathbf{650}}}{6\sqrt{5}M_*} \sum_{i,j=1}^3 \left[ (-3h_{ij}^a - 3h_{ij}'^s) \left\{ 2Q_L^i (U_L^c)^j H_u \right. \right. \\
& \left. \left. + 2Q_L^i (D_L^c)^j H_d + 2L_L^i (E_L^c)^j H_d + 2L_L^i (N_L^c)^j H_u \right\} \right], \quad (226)
\end{aligned}$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$\begin{aligned}
-\mathcal{L} \supset & \alpha' \frac{\tilde{v}'_{\mathbf{650}} F_T}{6\sqrt{5}M_*^2} \sum_{i,j=1}^3 \left[ (-3y_{ij}^a - 3y_{ij}'^s) \left\{ 2\tilde{Q}_L^i (\tilde{U}_L^c)^j H_u \right. \right. \\
& \left. \left. + 2\tilde{Q}_L^i (\tilde{D}_L^c)^j H_d + 2\tilde{L}_L^i (\tilde{E}_L^c)^j H_d + 2\tilde{L}_L^i (\tilde{N}_L^c)^j H_u \right\} \right]. \quad (227)
\end{aligned}$$

### 5.7. $E_6$ To $SU(5) \times U(1) \times SU(2)_X$

The breaking of  $E_6$  into  $SU(5) \times U(1) \times SU(2)_X$  can be realized via the VEVs of  $\mathbf{78}$  and  $\mathbf{650}$  dimensional representations. The  $(\mathbf{35}, \mathbf{1})$  component VEVs of the  $\mathbf{78}$  dimensional representation that break  $SU(6) \times SU(2)_X$  gauge group to  $SU(5) \times U(1) \times SU(2)_X$  reads

$$\langle \mathbf{78} \rangle_{(\mathbf{35}, \mathbf{1})} = \frac{\hat{v}_{\mathbf{78}}}{2\sqrt{15}} \text{diag}(\underbrace{1, 1, 1, 1, 1, -5}_2, \underbrace{2, \dots, 2}_{10}, \underbrace{-4, \dots, -4}_5), \quad (228)$$

with normalization factor  $c = 3$ . The  $(\mathbf{35}, \mathbf{1})$  component VEVs of the **650** dimensional representation that break  $SU(6) \times SU(2)_X$  gauge group to  $SU(5) \times U(1) \times SU(2)_X$  reads

$$\langle \mathbf{650} \rangle_{(\mathbf{35}, \mathbf{1})} = \frac{\hat{v}_{\mathbf{650}}}{\sqrt{30}} \text{diag}(\underbrace{1, 1, 1, 1, 1, -5}_2, \underbrace{-1, \dots, -1}_{10}, \underbrace{2, \dots, 2}_5), \quad (229)$$

with normalization factor  $c = 3$ .

After **78** dimensional Higgs acquire VEVs  $\langle \mathbf{78} \rangle_{(\mathbf{35}, \mathbf{1})}$ , the new contributions to superpotential

$$\begin{aligned} W &\supset \frac{\hat{v}_{\mathbf{78}}}{2\sqrt{15}M_*} \sum_{i,j=1}^3 \left( -3h_{ij}^a F_{(\mathbf{10}, \mathbf{1})}^i F_{(\mathbf{5}, \mathbf{2})}^j H_{(\mathbf{5}, \mathbf{2})} + 6h_{ij}^a F_{(\mathbf{5}, \mathbf{2})}^i F_{(\mathbf{1}, \mathbf{2})}^j H_{(\mathbf{5}, \mathbf{1})} \right), \\ &+ \frac{\hat{v}_{\mathbf{78}}}{2\sqrt{15}M_*} \sum_{i,j=1}^3 \left( 4h_{ij}' F_{(\mathbf{10}, \mathbf{1})}^i F_{(\mathbf{10}, \mathbf{1})}^j H_{(\mathbf{5}, \mathbf{1})} - 10h_{ij}' F_{(\mathbf{10}, \mathbf{1})}^i F_{(\mathbf{5}, \mathbf{2})}^j H_{(\mathbf{5}, \mathbf{2})} \right. \\ &\quad \left. + 8h_{ij}' F_{(\mathbf{5}, \mathbf{2})}^i F_{(\mathbf{1}, \mathbf{2})}^j H_{(\mathbf{5}, \mathbf{1})} \right), \\ &\supset \frac{\hat{v}_{\mathbf{78}}}{2\sqrt{15}M_*} \sum_{i,j=1}^3 \left[ (-3h_{ij}^a - 10h_{ij}') \{ Q_L^i (D_L^c)^j h_d + L_L^i (E_L^c)^j h_d \} \right], \\ &+ \frac{\hat{v}_{\mathbf{78}}}{2\sqrt{15}M_*} \sum_{i,j=1}^3 \left[ 8h_{ij}' Q_L^i (U_L^c)^j h_u + (6h_{ij}^a + 8h_{ij}') L_L^i (N_L^c)^j h_u \right], \quad (230) \end{aligned}$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$\begin{aligned} -\mathcal{L} &\supset \alpha' \frac{\hat{v}_{\mathbf{78}} F_T}{2\sqrt{15}M_*^2} \sum_{i,j=1}^3 \left[ (-3y_{ij}^a - 10y_{ij}') \{ \tilde{Q}_L^i (\tilde{D}_L^c)^j h_d + \tilde{L}_L^i (\tilde{E}_L^c)^j h_d \} \right], \\ &+ \alpha' \frac{\hat{v}_{\mathbf{78}} F_T}{2\sqrt{15}M_*^2} \sum_{i,j=1}^3 \left[ 8y_{ij}' \tilde{Q}_L^i (\tilde{U}_L^c)^j h_u + (6y_{ij}^a + 8y_{ij}') \tilde{L}_L^i (\tilde{N}_L^c)^j h_u \right] \quad (231) \end{aligned}$$

After **650** dimensional Higgs acquire VEVs  $\langle \mathbf{650} \rangle_{(\mathbf{35}, \mathbf{1})}$ , the new contributions to superpotential

$$W \supset \frac{\hat{v}_{\mathbf{650}}}{\sqrt{30}M_*} \sum_{i,j=1}^3 6h_{i,j=1}^a F_{(\mathbf{5}, \mathbf{2})}^i F_{(\mathbf{1}, \mathbf{2})}^j H_{(\mathbf{5}, \mathbf{1})},$$



$$\begin{aligned}
& + \frac{\hat{v}_{650}}{\sqrt{30}M_*} \sum_{i,j=1}^3 \left( h_{ij}'^s F_{(\mathbf{10},\mathbf{1})}^i F_{(\mathbf{10},\mathbf{1})}^j H_{(\mathbf{5},\mathbf{1})} \right. \\
& \quad \left. + 2h_{ij}'^s F_{(\mathbf{10},\mathbf{1})}^i F_{(\mathbf{5},\mathbf{2})}^j H_{(\mathbf{5},\mathbf{2})} - 4h_{ij}'^s F_{(\mathbf{5},\mathbf{2})}^i F_{(\mathbf{1},\mathbf{2})}^j H_{(\mathbf{5},\mathbf{1})} \right) , \\
& + \frac{\hat{v}_{650}}{\sqrt{30}M_*} \sum_{i,j=1}^3 \left( -2h_{ij}''^s F_{(\mathbf{10},\mathbf{1})}^i F_{(\mathbf{10},\mathbf{1})}^j H_{(\mathbf{5},\mathbf{1})} \right. \\
& \quad \left. + 2h_{ij}''^s F_{(\mathbf{10},\mathbf{1})}^i F_{(\mathbf{5},\mathbf{2})}^j H_{(\mathbf{5},\mathbf{2})} - 4h_{ij}''^s F_{(\mathbf{5},\mathbf{2})}^i F_{(\mathbf{1},\mathbf{2})}^j H_{(\mathbf{5},\mathbf{1})} \right) , \\
& \supset \frac{\hat{v}_{650}}{\sqrt{30}M_*} \sum_{i,j=1}^3 \left[ (2h_{ij}'^s - 4h_{ij}''^s) Q_L^i (U_L^c)^j h_u \right. \\
& \quad \left. + (6h_{ij}^a - 4h_{ij}'^s - 4h_{ij}''^s) L_L^i (N_L^c)^j h_u \right] , \\
& + \frac{\hat{v}_{650}}{\sqrt{30}M_*} \sum_{i,j=1}^3 \left[ (2h_{ij}'^s + 2h_{ij}''^s) \{ Q_L^i (D_L^c)^j h_d + L_L^i (E_L^c)^j h_d \} \right] , \quad (232)
\end{aligned}$$

while the new contributions to supersymmetry breaking soft trilinear terms

$$\begin{aligned}
-\mathcal{L} & \supset \alpha' \frac{\hat{v}_{650} F_T}{\sqrt{30}M_*^2} \sum_{i,j=1}^3 \left[ (2y_{ij}'^s - 4y_{ij}''^s) \tilde{Q}_L^i (\tilde{U}_L^c)^j h_u \right. \\
& \quad \left. + (6y_{ij}^a - 4y_{ij}'^s - 4y_{ij}''^s) \tilde{L}_L^i (\tilde{N}_L^c)^j h_u \right] , \\
& + \alpha' \frac{\hat{v}_{650} F_T}{\sqrt{30}M_*^2} \sum_{i,j}^3 \left[ (2y_{ij}'^s + 2y_{ij}''^s) \{ \tilde{Q}_L^i (\tilde{D}_L^c)^j h_d + \tilde{L}_L^i (\tilde{E}_L^c)^j h_d \} \right] . \quad (233)
\end{aligned}$$

## 6. Scalar and Gaugino Mass Relations

In order to study the scalar and gaugino mass relations [37, 67] that are invariant under one-loop renormalization group running, we need to know the renormalization group equations (RGEs) of the supersymmetry breaking scalar masses and gaugino masses. For simplicity, we only consider the one-loop RGE running since the two-loop RGE running effects are small [35]. In particular, for the first two generations, we can neglect the contributions from the Yukawa coupling terms and trilinear soft terms, and then the RGEs

for the scalar masses are [66]

$$16\pi^2 \frac{dm_{\tilde{Q}_j}^2}{dt} = -\frac{32}{3}g_3^2 M_3^2 - 6g_2^2 M_2^2 - \frac{2}{15}g_1^2 M_1^2 + \frac{1}{5}g_1^2 S, \quad (234)$$

$$16\pi^2 \frac{dm_{\tilde{U}_j^c}^2}{dt} = -\frac{32}{3}g_3^2 M_3^2 - \frac{32}{15}g_1^2 M_1^2 - \frac{4}{5}g_1^2 S, \quad (235)$$

$$16\pi^2 \frac{dm_{\tilde{D}_j^c}^2}{dt} = -\frac{32}{3}g_3^2 M_3^2 - \frac{8}{15}g_1^2 M_1^2 + \frac{2}{5}g_1^2 S, \quad (236)$$

$$16\pi^2 \frac{dm_{\tilde{L}_j}^2}{dt} = -6g_2^2 M_2^2 - \frac{6}{5}g_1^2 M_1^2 - \frac{3}{5}g_1^2 S, \quad (237)$$

$$16\pi^2 \frac{dm_{\tilde{E}_j^c}^2}{dt} = -\frac{24}{5}g_1^2 M_1^2 + \frac{6}{5}g_1^2 S, \quad (238)$$

where  $j = 1, 2$ , and  $t = \ln \mu$  and  $\mu$  is the renormalization scale. Also,  $S$  is given by

$$\begin{aligned} S &= \text{Tr}[Y_{\phi_i} m^2(\phi_i)] \\ &= m_{H_u}^2 - m_{H_d}^2 + \text{Tr}[M_{\tilde{Q}_i}^2 - M_{\tilde{L}_i}^2 - 2M_{\tilde{U}_i^c}^2 + M_{\tilde{D}_i^c}^2 + M_{\tilde{E}_i^c}^2]. \end{aligned} \quad (239)$$

The one-loop RGEs for gauge couplings  $g_i$  and gaugino masses  $M_i$  are

$$\frac{d}{dt}g_i = \frac{1}{16\pi^2}b_i g_i^3, \quad \frac{d}{dt}M_i = \frac{1}{8\pi^2}b_i g_i^2 M_i, \quad (240)$$

where  $g_1 \equiv \sqrt{5}g_Y/\sqrt{3}$ , and  $b_1$ ,  $b_2$  and  $b_3$  are one-loop beta functions for  $U(1)_Y$ ,  $SU(2)_L$ , and  $SU(3)_C$ , respectively. For the supersymmetric SM, we have

$$b_3 = -3, \quad b_2 = 1, \quad b_1 = \frac{33}{5}. \quad (241)$$

Therefore, we obtain

$$\begin{aligned} \frac{d}{dt} \left[ \frac{MSQj}{Y_{Q_j}} \right] &= \frac{d}{dt} \left[ \frac{MSUj}{Y_{U_j^c}} \right] = \frac{d}{dt} \left[ \frac{MSDj}{Y_{D_j^c}} \right] \\ &= \frac{d}{dt} \left[ \frac{MSLj}{Y_{L_j}} \right] = \frac{d}{dt} \left[ \frac{MSEj}{Y_{E_j^c}} \right], \end{aligned} \quad (242)$$

where

$$MSQj = 4m_{\tilde{Q}_j}^2 + \frac{32}{3b_3}M_3^2 + \frac{6}{b_2}M_2^2 + \frac{2}{15b_1}M_1^2, \quad (243)$$

$$MSUj = 4m_{\tilde{U}_j^c}^2 + \frac{32}{3b_3}M_3^2 + \frac{32}{15b_1}M_1^2, \quad (244)$$

$$MSDj = 4m_{\tilde{D}_j^c}^2 + \frac{32}{3b_3}M_3^2 + \frac{8}{15b_1}M_1^2, \quad (245)$$

$$MSLj = 4m_{\tilde{L}_j}^2 + \frac{6}{b_2}M_2^2 + \frac{6}{5b_1}M_1^2, \quad (246)$$

$$MSEj = 4m_{\tilde{E}_j^c}^2 + \frac{24}{5b_1}M_1^2. \quad (247)$$

In addition, we obtain the most general scalar and gaugino mass relations that are valid from the GUT scale to the electroweak scale under one-loop RGE running for the first two families

$$\gamma_{Q_j} \frac{MSQj}{Y_{Q_j}} + \gamma_{U_j^c} \frac{MSUj}{Y_{U_j^c}} + \gamma_{D_j^c} \frac{MSDj}{Y_{D_j^c}} + \gamma_{L_j} \frac{MSLj}{Y_{L_j}} + \gamma_{E_j^c} \frac{MSEj}{Y_{E_j^c}} = C_o, \quad (248)$$

where  $C_o$  denotes the invariant constant under one-loop RGE running, and  $\gamma_{Q_j}$ ,  $\gamma_{U_j^c}$ ,  $\gamma_{D_j^c}$ ,  $\gamma_{L_j}$ , and  $\gamma_{E_j^c}$  are real or complex numbers that satisfy

$$\gamma_{Q_j} + \gamma_{U_j^c} + \gamma_{D_j^c} + \gamma_{L_j} + \gamma_{E_j^c} = 0. \quad (249)$$

In short, we can obtain the scalar and gaugino mass relations that are valid from the GUT scale to the electroweak scale at one loop. Such relations will be useful to distinguish between the mSUGRA and GmSUGRA scenarios.

The scalar and gaugino mass relations can be simplified by the scalar and gaugino mass relations at the GUT scale. Because the higher-dimensional operators can contribute to gauge kinetic functions after GUT symmetry breaking, the SM gauge couplings may not be unified at the GUT scale. Thus, we will have two contributions to the gaugino masses at the GUT scale: the universal gaugino masses as in the mSUGRA, and the non-universal gaugino masses due to the higher-dimensional operators. In particular, for the scenarios studied in Refs. [17, 18, 19, 20, 21, 22, 23] where the universal

gaugino masses are assumed to be zero, *i.e.*,  $M_i/\alpha_i = a_i M'_{1/2}$ , we obtain the gaugino mass relation at one loop [36]

$$\frac{M_3}{a_3 \alpha_3} = \frac{M_2}{a_2 \alpha_2} = \frac{M_1}{a_1 \alpha_1} . \quad (250)$$

We can calculate the scalar and gaugino mass relations in the mSUGRA and GmSUGRA scenarios, and compare them in different cases.

The RGE running invariant combinations in  $SU(5)$ ,  $SO(10)$ , Pati-Salam model had been discussed in our previous works [37]. We only discuss here the  $SU(3)_C \times SU(3)_L \times SU(3)_R$  case from  $E_6$  breaking.

We consider the following  $E_6$  gauge symmetry breaking chain

$$\begin{aligned} E_6 &\rightarrow SU(3)_C \times SU(3)_L \times SU(3)_R \\ &\rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ &\rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y . \end{aligned} \quad (251)$$

Other symmetry breaking chains can be discussed similarly.

Let us explain our convention. We denote the gauge couplings for the  $SU(2)_L$ ,  $SU(2)_R$ ,  $SU(3)_L$ ,  $SU(3)_R$ ,  $U(1)_{B-L}$  and  $SU(3)_C$  gauge symmetries as  $g_{2L}$ ,  $g_{2R}$ ,  $g_{3L}$ ,  $g_{3R}$ ,  $\tilde{g}_{B-L}$  (or traditional  $g_{B-L}$ ), and  $g_3$ , respectively. We denote the gaugino masses for the  $SU(2)_L$ ,  $SU(2)_R$ ,  $SU(3)_L$ ,  $SU(3)_R$ ,  $U(1)_{B-L}$ , and  $SU(3)_C$  gauge symmetries as  $M_{2L}$ ,  $M_{2R}$ ,  $M_{3L}$ ,  $M_{3R}$ ,  $M_{B-L}$ , and  $M_3$ , respectively. We denote the one-loop beta functions for the  $SU(2)_L$ ,  $SU(2)_R$ ,  $SU(3)_L$ ,  $SU(3)_R$ ,  $U(1)_{B-L}$ , and  $SU(3)_C$  gauge symmetries as  $b_{2L}$ ,  $b_{2R}$ ,  $b_{3L}$ ,  $b_{3R}$ ,  $\tilde{b}_{B-L}$  and  $b_4$ , respectively. In addition, we denote the universal supersymmetry breaking scale as  $M_S$ , the  $SU(2)_R \times U(1)_{B-L}$  gauge symmetry breaking scale as  $M_{LR}$ , and the  $SU(3)_C \times SU(3)_L \times SU(3)_R$  gauge symmetry breaking scale as  $M_{33}$ . Also, we denote the  $U(1)_{B-L}$  charge for the particle  $\phi_i$  as  $Y_{\phi_i}^{B-L}$ .

Neglecting the Yukawa coupling terms and trilinear soft terms, we obtain the RGEs for the scalar masses of the first two generations in the gauge group  $SU(3)_C \times SU(3)_L \times SU(3)_R$

$$16\pi^2 \frac{dm_{\tilde{X}_L}^2}{dt} = 4\pi^2 \frac{d}{dt} \left[ -\frac{32}{3b_3} M_3^2 - \frac{32}{3b_{3L}} M_{3L}^2 \right] , \quad (252)$$

$$16\pi^2 \frac{dm_{\tilde{X}_R}^2}{dt} = 4\pi^2 \frac{d}{dt} \left[ -\frac{32}{3b_3} M_3^2 - \frac{32}{3b_{3R}} M_{3R}^2 \right] , \quad (253)$$

$$16\pi^2 \frac{dm_{\tilde{N}}^2}{dt} = 4\pi^2 \frac{d}{dt} \left[ -\frac{32}{3b_{3L}} M_{3L}^2 - \frac{32}{3b_{3R}} M_{3R}^2 \right] , \quad (254)$$

which gives

$$\frac{d}{dt} \left[ m_{\tilde{X}_L}^2 + \frac{8}{3b_3} M_3^2 + \frac{8}{3b_{3L}} M_{3L}^2 \right] = 0 , \quad (255)$$

$$\frac{d}{dt} \left[ m_{\tilde{X}_L}^2 + \frac{8}{3b_3} M_3^2 + \frac{8}{3b_{3R}} M_{3R}^2 \right] = 0 \quad (256)$$

$$\frac{d}{dt} \left[ m_{\tilde{N}}^2 + \frac{8}{3b_{3L}} M_{3L}^2 + \frac{8}{3b_{3R}} M_{3R}^2 \right] = 0. \quad (257)$$

The RGEs of the scalar masses for the first two generations in the left right model  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  are

$$\begin{aligned} 16\pi^2 \frac{dm_{\tilde{Q}_j}^2}{dt} &= -\frac{32}{3} g_3^2 M_3^2 - 6g_{2L}^2 M_{2L}^2 - \frac{1}{3} \tilde{g}_{B-L}^2 M_{B-L}^2 + \frac{1}{2} \tilde{g}_{B-L}^2 S' , \\ 16\pi^2 \frac{dm_{\tilde{U}_j^c, \tilde{D}_j^c}^2}{dt} &= -\frac{32}{3} g_3^2 M_3^2 - 6g_{2R}^2 M_{2R}^2 - \frac{1}{3} \tilde{g}_{B-L}^2 M_{B-L}^2 - \frac{1}{2} \tilde{g}_{B-L}^2 S' , \\ 16\pi^2 \frac{dm_{\tilde{L}_j}^2}{dt} &= -6g_{2L}^2 M_{2L}^2 - 3\tilde{g}_{B-L}^2 M_{B-L}^2 - \frac{3}{2} \tilde{g}_{B-L}^2 S' , \\ 16\pi^2 \frac{dm_{\tilde{E}_j^c}^2}{dt} &= -6g_{2R}^2 M_{2R}^2 - 3\tilde{g}_{B-L}^2 M_{B-L}^2 + \frac{3}{2} \tilde{g}_{B-L}^2 S' , \end{aligned} \quad (258)$$

where

$$S' = Tr[Y_{\phi_i}^{B-L} m^2(\phi_i)] . \quad (259)$$

We consider the following linear combination of the squared scalar masses

$$\begin{aligned} & 16\pi^2 \frac{d}{dt} \left( m_{\tilde{U}_j^c}^2 + m_{\tilde{E}_j^c}^2 - 2m_{\tilde{Q}_j}^2 \right) \\ &= 4\pi^2 \frac{d}{dt} \left[ \frac{32}{3b_3} M_3^2 + \frac{12}{b_{2L}} M_{2L}^2 - \frac{20}{3b_1} M_1^2 \right] \quad \text{for } M_S < \mu < M_{LR} \\ &= 4\pi^2 \frac{d}{dt} \left[ \frac{32}{3b_3} M_3^2 + \frac{12}{b_{2L}} M_{2L}^2 - \frac{12}{b_{2R}} M_{2R}^2 - \frac{8}{3\tilde{b}_{B-L}} M_{B-L}^2 \right] \\ &\quad \text{for } M_{LR} < \mu < M_{33} \\ &= 4\pi^2 \frac{d}{dt} \left[ -\frac{64}{3b_{3R}} M_{3R}^2 + \frac{32}{3b_{3L}} M_{3L}^2 + \frac{32}{3b_3} M_3^2 \right] \quad \text{for } M_{33} < \mu < M_U . \end{aligned} \quad (260)$$

From the RGE invariant combinations, we obtain the one-loop exact scalar and gaugino mass relations from the GUT scale to the electroweak scale

$$\begin{aligned}
4 \left( m_{\tilde{U}_j^c}^2 + m_{\tilde{E}_j^c}^2 - 2m_{\tilde{Q}_j}^2 \right) - \frac{32M_3^2}{3b_3} - \frac{12M_{2L}^2}{b_{2L}} + \frac{20M_1^2}{3b_1} &= C_o^1, \\
4 \left( m_{\tilde{U}_j^c}^2 + m_{\tilde{E}_j^c}^2 - 2m_{\tilde{Q}_j}^2 \right) - \frac{32M_3^2}{3b_3} - \frac{12M_{2L}^2}{b_{2L}} + \frac{12M_{2R}^2}{b_{2R}} + \frac{8M_{B-L}^2}{3\tilde{b}_{B-L}} &= C_o^2, \\
4 \left( m_{\tilde{U}_j^c}^2 + m_{\tilde{E}_j^c}^2 - 2m_{\tilde{Q}_j}^2 \right) - \frac{32M_3^2}{3b_3} + \frac{64M_{3R}^2}{3b_{3R}} - \frac{32M_{3L}^2}{3b_{3L}} &= C_o^3. \quad (261)
\end{aligned}$$

The differences between the constants  $C_o^1$  and  $C_o^2$  and between the constants  $C_o^2$  and  $C_o^3$  are the threshold contributions from the extra particles due to gauge symmetry breaking. Thus, the three constants can be determined by matching the threshold contributions at the symmetry breaking scales. The difference between  $C_o^2$  and  $C_o^3$  is

$$C_o^2 - C_o^3 = \left( \frac{12}{b_{2R}} - \frac{64}{3b_{3R}} \right) M_{3R}^2 + \left( \frac{32}{3b_{3L}} - \frac{12}{b_{2L}} \right) M_{3L}^2 + \frac{8}{3\tilde{b}_{B-L}} M_{B-L}^2, \quad (262)$$

while the difference between  $C_o^1$  and  $C_o^2$  is

$$C_o^1 - C_o^2 = -\frac{12}{b_{2R}} M_{2R}^2 - \frac{8}{3\tilde{b}_{B-L}} M_{B-L}^2 + \frac{20}{3b_1} M_1^2. \quad (263)$$

At the  $SU(3)_C \times SU(3)_L \times SU(3)_R$  unification scale  $M_{33}$ , we have

$$\frac{1}{g_{B-L}^2} = \frac{1}{g_{3L}^2} + \frac{1}{g_{3R}^2}. \quad (264)$$

For mSUGRA with universal gaugino and scalar masses, we have

$$\frac{M_3}{g_3^2} = \frac{M_{2L}}{g_{2L}^2} = \frac{M_{2R}}{g_{2R}^2} = \frac{M_{3L}}{g_{3L}^2} = \frac{M_{3R}}{g_{3R}^2}. \quad (265)$$

Thus, we can get the scalar and gaugino mass relations in supersymmetric Standard Model

$$\begin{aligned}
&4 \left( m_{\tilde{U}_j^c}^2 + m_{\tilde{E}_j^c}^2 - 2m_{\tilde{Q}_j}^2 \right) - \frac{32}{3b_3} M_3^2 - \frac{12}{b_{2L}} M_2^2 + \frac{20}{3b_1} M_1^2 \\
&= \left( 2\frac{8}{3\tilde{b}_{B-L}} - \frac{32}{3b_{3L}} \right) \frac{M_3^2(\mu)}{g_3^4(\mu)} g_3^4(M_{33}) + \frac{20}{3b_1} \frac{M_1^2(\mu)}{g_1^4(\mu)} g_1^4(M_{LR}) \\
&\quad - \left( \frac{12}{b_{2R}} g_{2R}^4(M_{LR}) + \frac{8}{3\tilde{b}_{B-L}} g_{B-L}^4(M_{LR}) \right) \frac{M_3^2(\mu)}{g_3^4(\mu)}. \quad (266)
\end{aligned}$$

Here we use the fact that  $b_3 = b_{3L} = b_{3R}$  for  $(M_{E_6} > \mu > M_{33})$  as well as  $b_{2L} = b_{2R}$  for  $(M_{33} > \mu > M_{LR})$ . If we know the low energy sparticle spectrum at the LHC and ILC and  $g_1^2(M_{LR})$  from the RGE running, we can get the coefficients

$$c = \left( \frac{16}{3\tilde{b}_{B-L}} - \frac{32}{b_3} \right) g_3^4(M_{PS}) - \left( \frac{12}{b_{2R}} g_{2R}^4(M_{LR}) + \frac{8}{3\tilde{b}_{B-L}} g_{B-L}^4(M_{LR}) \right), \quad (267)$$

by fitting the experimental data.

For GmSUGRA with non-universal gaugino and scalar masses, we consider the Higgs field in the **650** representation whose singlet component  $(\mathbf{1}, \mathbf{1}, \mathbf{1})$  acquires VEVs. To give mass to the gluino, we require that the universal gaugino mass be non-zero. From Eq. (75), we obtain

$$m_{\tilde{E}_j^c}^2 + m_{\tilde{U}_j^c}^2 - 2m_{\tilde{Q}_j}^2 = -\frac{\sqrt{2}}{2}(\beta'^{\mathbf{650}} v_{\mathbf{650}}) \frac{|F_S|^2}{M_*^3}. \quad (268)$$

Thus, the constant combination in the supersymmetric Standard Model is

$$\begin{aligned} & 4 \left( m_{\tilde{U}_j^c}^2 + m_{\tilde{E}_j^c}^2 - 2m_{\tilde{Q}_j}^2 \right) - \frac{32}{3b_3} M_3^2 - \frac{12}{b_{2L}} M_{2L}^2 + \frac{20}{3b_1} M_1^2 \\ &= -\frac{\sqrt{2}}{2}(\beta'^{\mathbf{650}} v_{\mathbf{650}}) \frac{|F_S|^2}{M_*^3} + \frac{20}{3b_1} \frac{M_1^2(\mu)}{g_1^4(\mu)} g_1^4(M_{LR}) \\ &- \left( \frac{12}{b_{2R}} g_{2R}^4(M_{LR}) \frac{M_{2R}^2(\mu)}{g_{2R}^4(\mu)} + \frac{8}{3\tilde{b}_{B-L}} g_{B-L}^4(M_{LR}) \frac{M_3^2(\mu)}{g_3^4(\mu)} \right) \\ &+ \left( \frac{16}{3\tilde{b}_{B-L}} - \frac{32}{3b_{3L}} \right) \frac{M_3^2(\mu)}{g_3^4(\mu)} g_3^4(M_{33}). \end{aligned} \quad (269)$$

Therefore, the scalar and gaugino mass relations in mSUGRA are different from those in GmSUGRA. Similar discussions can be used for other  $E_6$  gauge symmetry breaking chains and we will not present here.

## 7. Conclusions

In the GmSUGRA scenario with the higher-dimensional operators containing the GUT Higgs fields, we systematically studied the supersymmetry breaking scalar masses, SM fermion Yukawa coupling terms, and trilinear

soft terms in the  $E_6$  model where the gauge symmetry is broken down to the  $SO(10) \times U(1)$  gauge symmetry,  $SU(3)_C \times SU(3)_L \times SU(3)_R$  gauge symmetry,  $SU(6) \times SU(2)_a$  ( $a = L, R, X$ ) gauge symmetry, flipped  $SU(5)$  gauge symmetry. In addition, we considered the scalar and gaugino mass relations, which can be preserved from the GUT scale to the electroweak scale under one-loop RGE running, in the  $SU(3)_C \times SU(3)_L \times SU(3)_R$  model arising from the  $E_6$  model. With such relations, we may distinguish the mSUGRA and GmSUGRA scenarios if we can measure the supersymmetric particle spectrum at the LHC and ILC. Thus, it provides us with another important window of opportunity at the Planck scale.

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